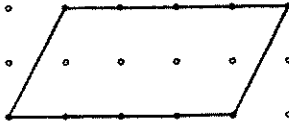
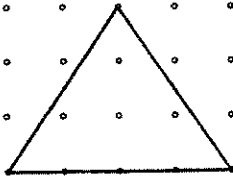


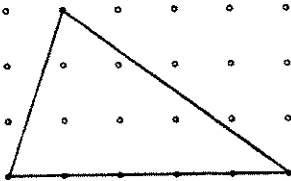
PARALLELOGRAM



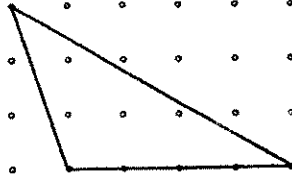
ISOSCELES TRIANGLE



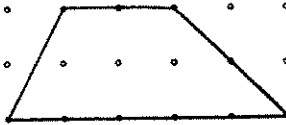
ACUTE SCALENE TRIANGLE



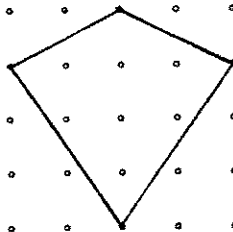
OBTUSE SCALENE TRIANGLE



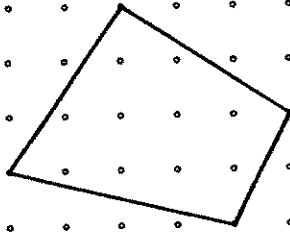
TRAPEZOID



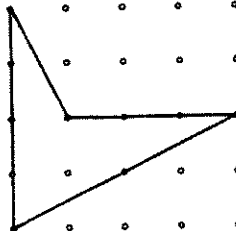
KITE



CONVEX QUADRILATERAL°

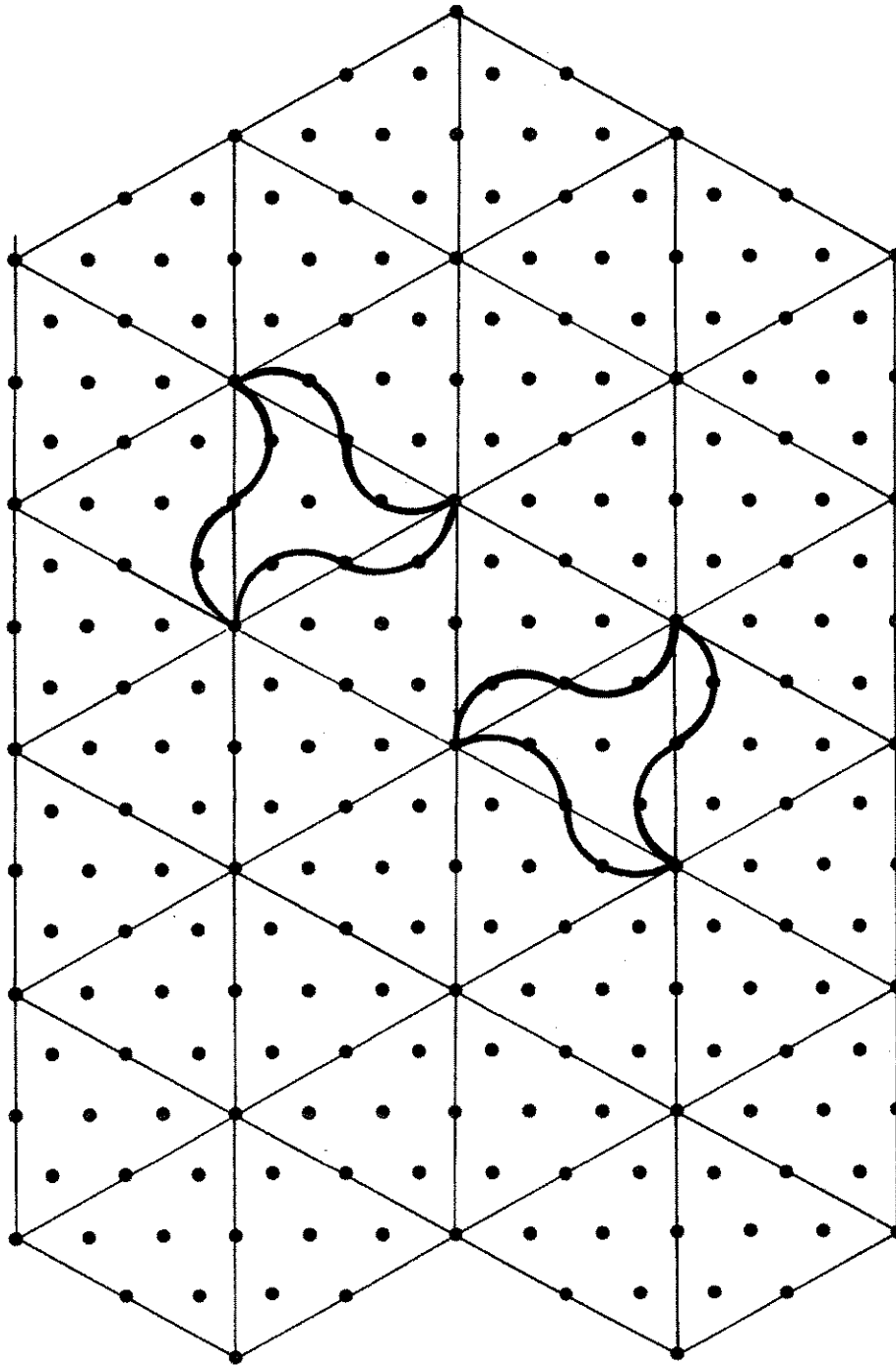


CONCAVE QUADRILATERAL



ACTIVITY 99: Free Flight

Use a circle template or a French curve, or draw around a penny to continue the pattern. Tessellate the page. Color your design with two, three, or four colors of your choice.



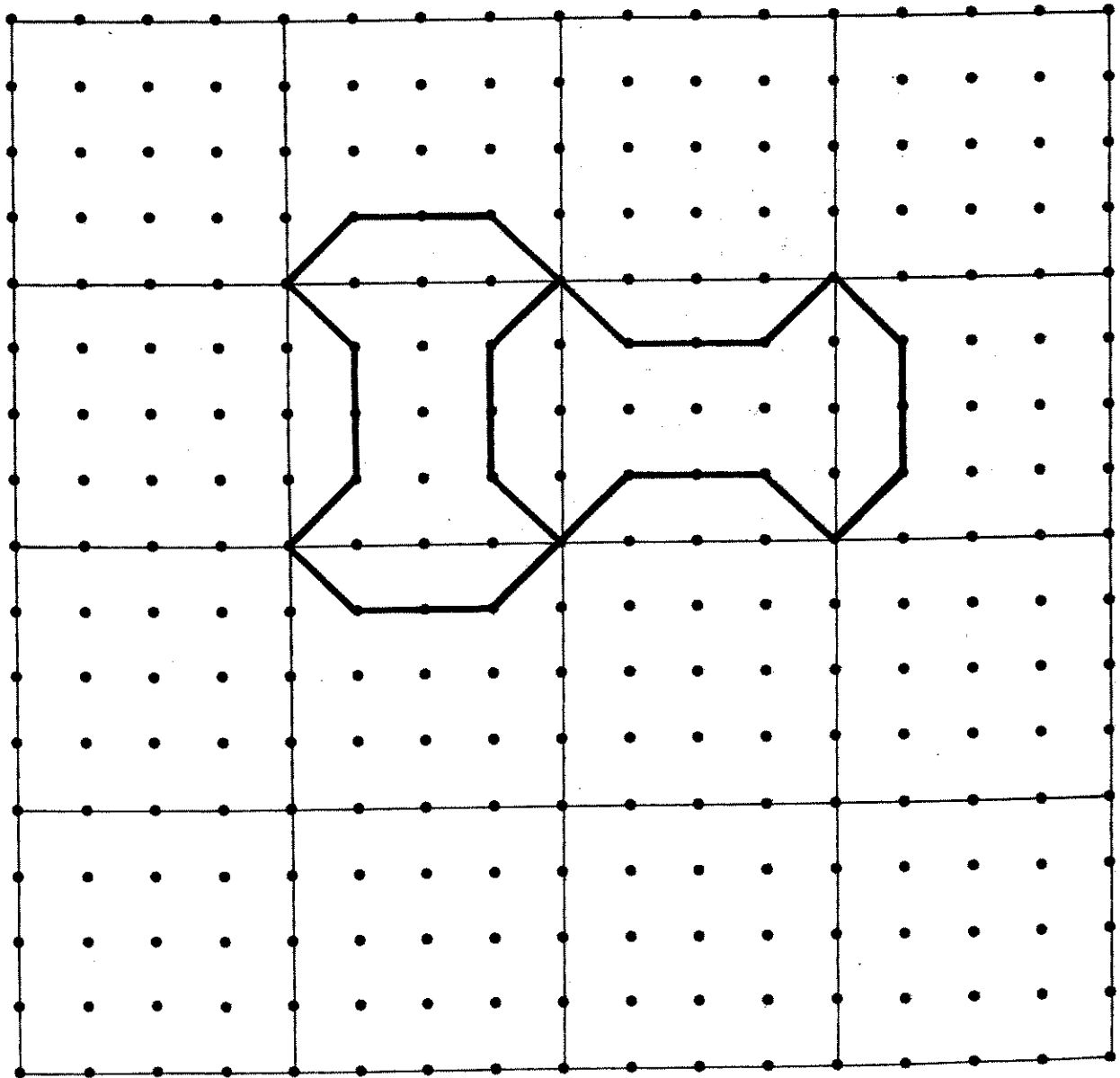
ACTIVITY 92: Bow Ties

Take a regular tessellation of squares. Remove the same congruent pieces from two opposite sides of the square region and attach these to the other two sides of the square region. The result is a bow tie. Complete the tiling below.

Is the perimeter of the square the same as the perimeter of the bow tie?

Is the area the same?

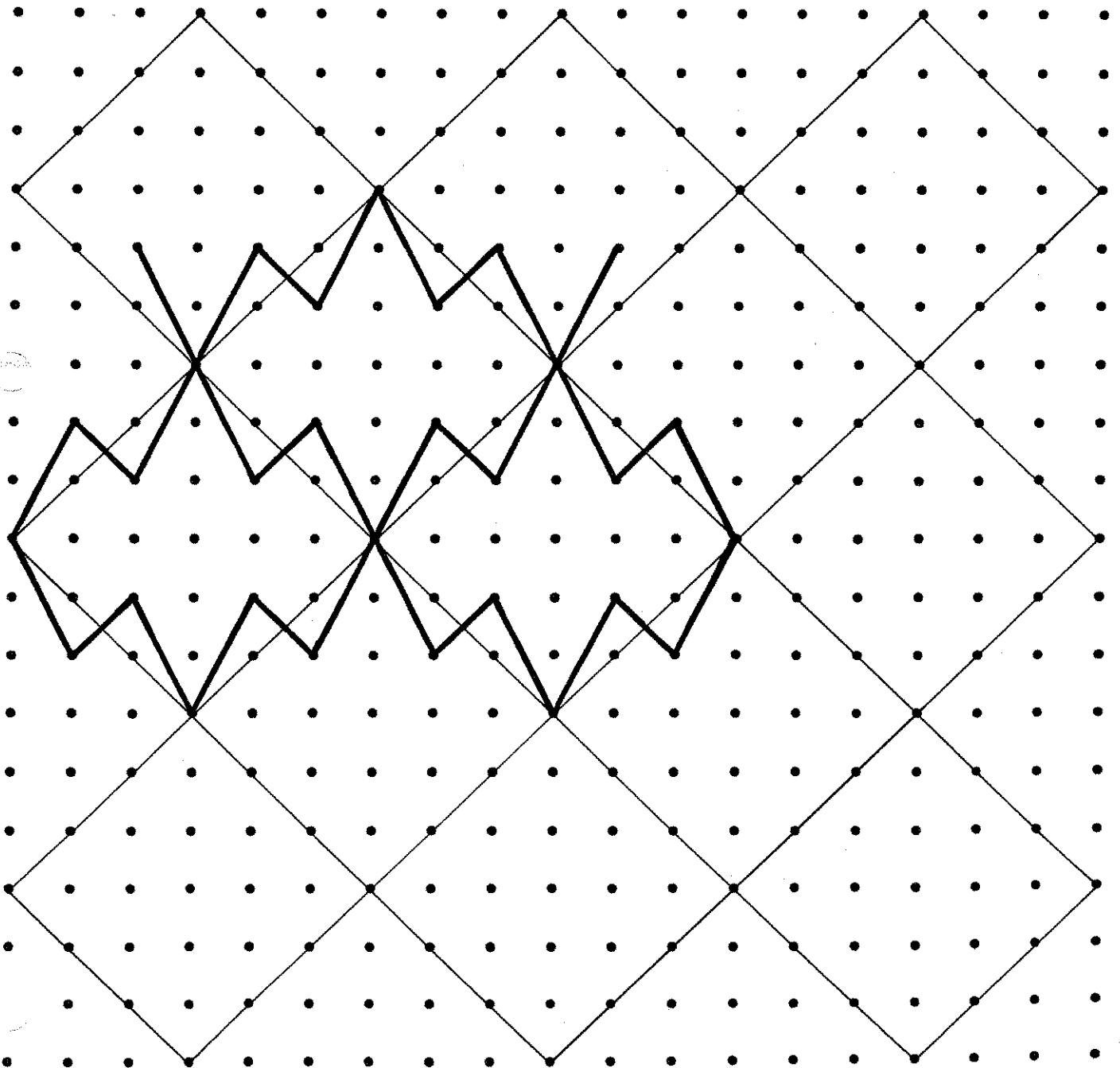
This design is based on the Alhambra tiles.



ACTIVITY 86: Bats

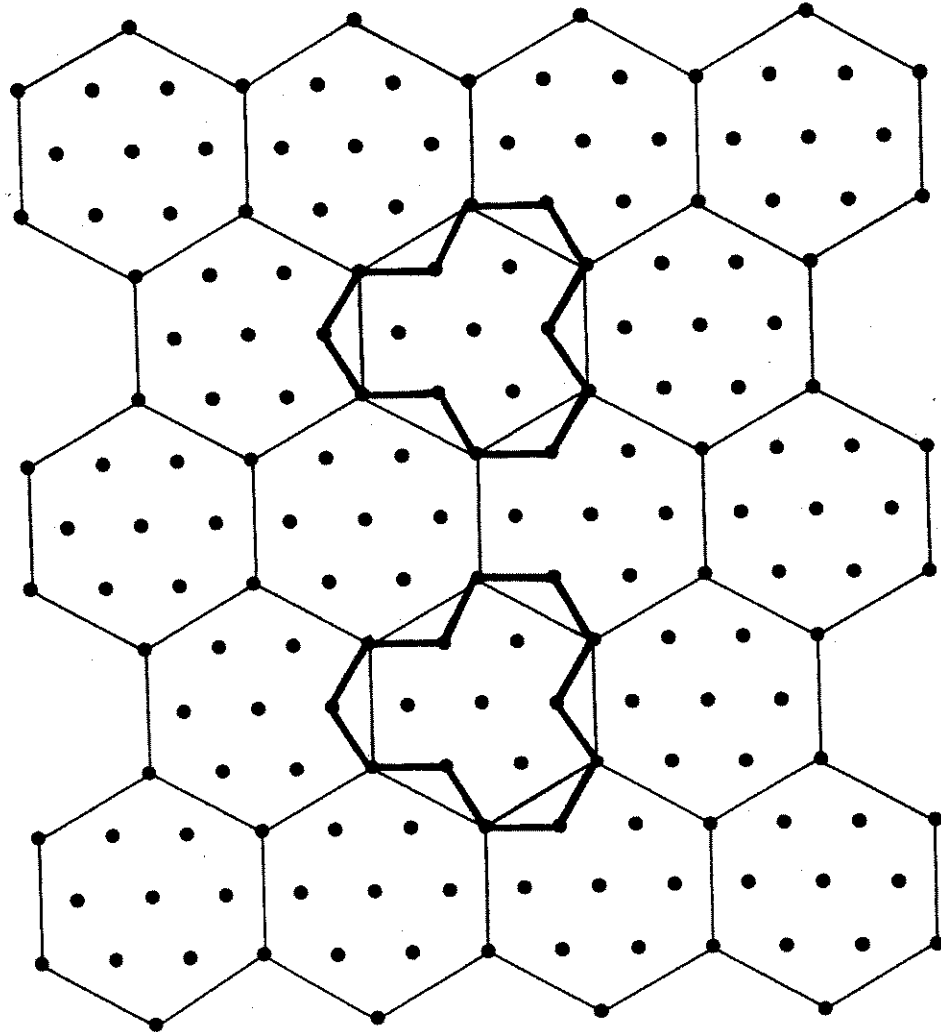
Study the tessellation that has been started below. Finish the tessellation as far as possible. Be able to describe how the tessellation was created from the square grid.

Is it possible to use just two colors (orange and black) to color the bats in such a way that no bat of one color is adjacent to a bat of the same color? This design is based on the Alhambra tiles.



ACTIVITY 104: Spinners


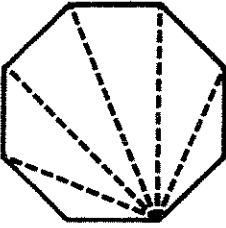

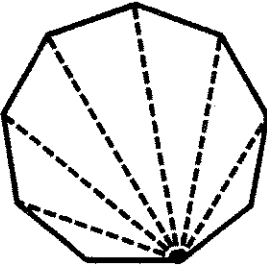

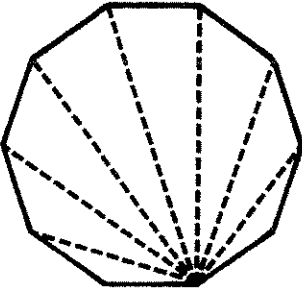
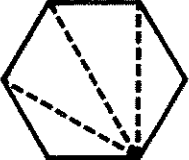
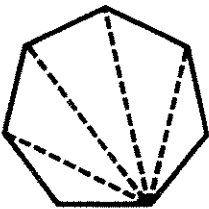
Use the regular tessellation of hexagons below. Remove three congruent pieces from the hexagon and attach each to an adjacent side of the hexagon so that congruent edges meet. The result is a spinner. Complete the tiling below.



ACTIVITY 6

In this activity you will find the degree measures of the interior angles of regular polygons. Fill in the blanks in Table 1. Use diagonals to help you.

Table 1

Regular Polygon	Number of Sides	Degree Measure of Each Interior Angle	Regular Polygon	Number of Sides	Degree Measure of Each Interior Angle
Equilateral Triangle 	3	60°	Octagon 		
Square 	4	90°	Nonagon 		
Pentagon 			Decagon 		
Hexagon 					
Heptagon 					

In Activity 4, you found that it is possible to tessellate a plane with squares. Notice that at vertex point *A* in Figure 9 (and all other vertex points of that tessellation) there are the vertices of 4 squares, and there are 4 angles about each vertex point. The measure of each interior angle of a square is 90° . Thus, the sum of the measures of all the angles about vertex point *A* (and all other vertex points of the tessellation) is

$$4 \times 90^\circ = 360^\circ.$$

Therefore, 4 squares will fit together at a vertex point without gaps or overlappings.

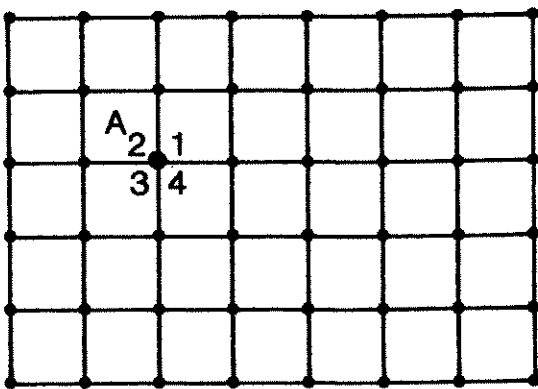


Figure 9
Squares

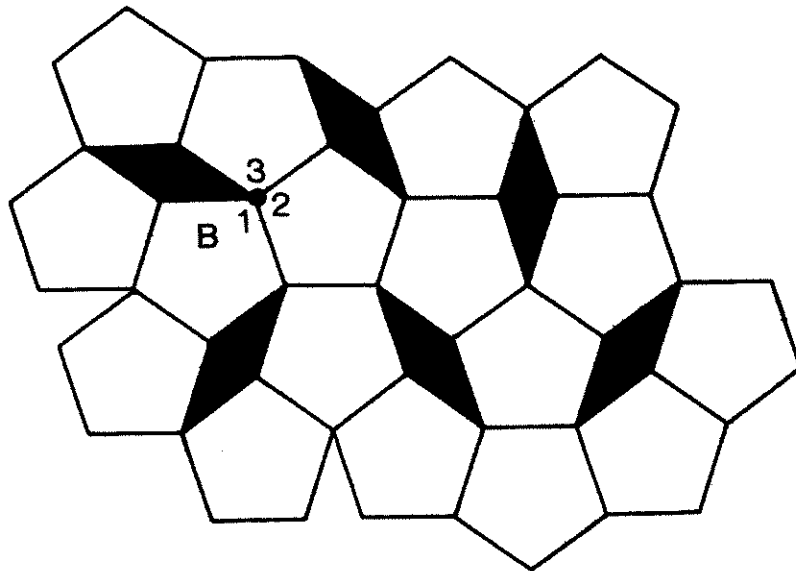


Figure 10
Regular Pentagons

It is not possible to tessellate a plane with regular pentagons. In Figure 10, the darkened regions represent gaps. At vertex point *B* (and all other vertex points of that tessellation), there are vertices of 3 pentagons. The measure

of each interior angle of a regular pentagon is 108° . Thus, the sum of the measures of the 3 interior angles of the pentagons at the vertex point B is

$$3 \times 108^\circ = 324^\circ.$$

Since 324° is less than 360° , we have gaps. If you were to put 4 regular pentagons at vertex point B , then the sum of the measures of the 4 interior angles of the pentagons would be

$$4 \times 108^\circ = 432^\circ,$$

and there would be an overlap. — Try it. Use a template or the pentagons you cut out for Activity 4.

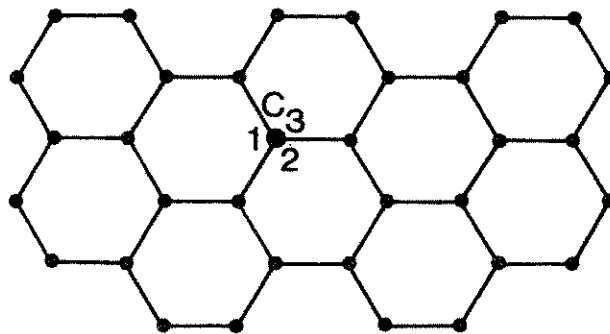


Figure 11
Regular Hexagons

It is possible to tessellate a plane with regular hexagons. At vertex point C in Figure 11 (and all vertex points of that tessellation), there are the vertices of 3 hexagons, and there are 3 angles about each vertex point. Each interior angle of a regular hexagon is 120° . Thus, the sum of the measures of all the angles at the vertex point C (and all vertex points of the tessellation) is exactly

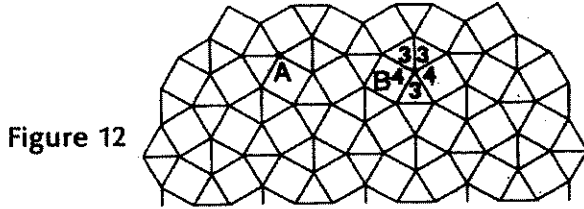
$$3 \times 120^\circ = 360^\circ.$$

6. SEMIREGULAR TESSELLATIONS

A regular tessellation of the plane covers the plane with repetitions of one particular regular polygon. It is quite natural to ask:

Is it possible to tessellate a plane using combinations of regular polygons?

Study Figure 12.



It shows a partial tessellation of the plane using a combination of equilateral triangles and squares.

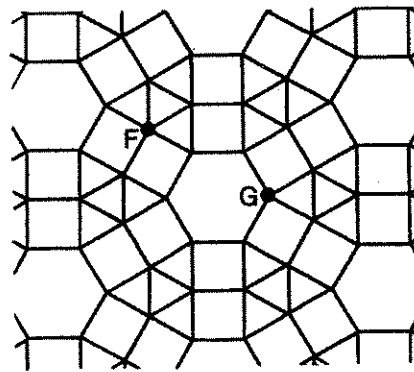
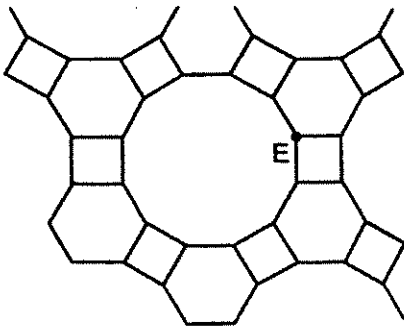
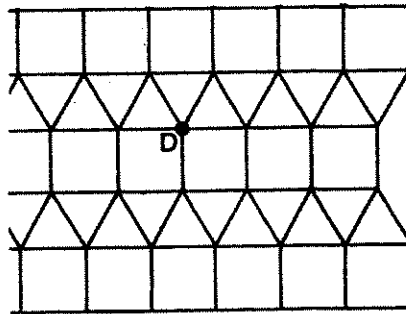
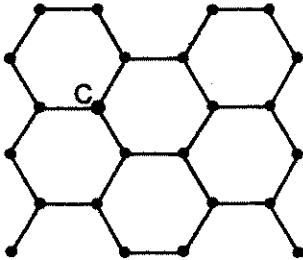
At each and every vertex point (notice vertex points *A* and *B* for example) of the tessellation, there are the vertices of 3 equilateral triangles and the vertices of 2 squares.

The sum of the measures of the angles at each vertex point of the tessellation is

$$(3 \times 60^\circ) + (2 \times 90^\circ) = 180^\circ + 180^\circ = 360^\circ.$$

equilateral triangles
squares

This result explains why the tessellation is possible.



ACTIVITY 26

In this activity, we will see why all convex quadrilaterals tessellate.

Make a copy of the quadrilateral in Figure 23 on tracing paper. Paste the tracing paper onto cardboard and cut out the quadrilateral. Mark the angles as shown in the figure. Use the quadrilateral as a tracing model.

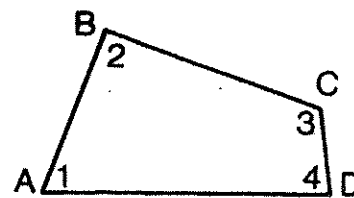


Figure 23

Study Figure 24. It is constructed as explained below.

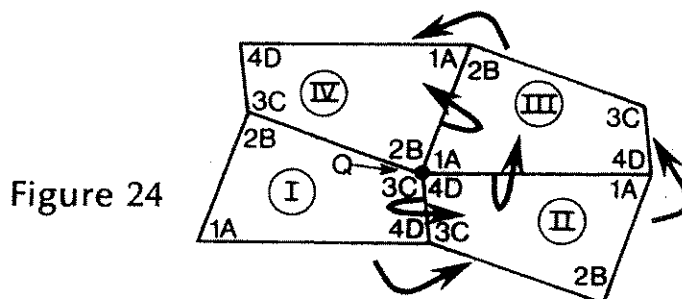


Figure 24

1. Begin with the quadrilateral $ABCD$ in the position marked I. Trace it. Rotate the quadrilateral about the midpoint of side CD . Trace it and label the angles in the quadrilateral in this position. Call this position II.

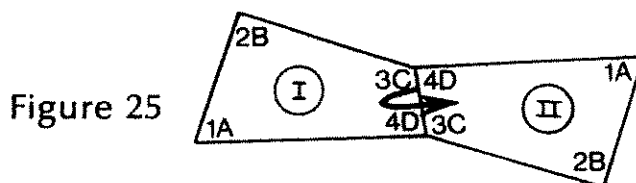


Figure 25

2. Next rotate the quadrilateral in position II about the midpoint of the side AD . Trace it and mark the angles in the quadrilateral in this position. Call this position III.
3. Now rotate the quadrilateral in position III about the midpoint of the side AB . Trace it and mark the angles in the quadrilateral in this position. Call this position IV.

In the resulting figure, all 4 quadrilaterals have a vertex in common at the point Q .

What is the sum of the measures of all the angles about vertex Q ? _____

Notice that all the original angles of the quadrilateral $ABCD$ occur at the vertex point Q .

Now continue the procedure described above and tessellate a plain sheet of paper. Keep marking the angles in each position of the quadrilateral as you proceed.

Do angles 1, 2, 3, 4 appear at each vertex point in the tessellation?

Yes _____ No _____

ACTIVITY 27

In this activity, we will see why all concave quadrilaterals tessellate. Make a copy of the concave quadrilateral in Figure 26 on tracing paper. Paste the tracing paper onto cardboard and cut out the quadrilateral. Mark the angles as shown in Figure 26. Use the quadrilateral as a tracing model.

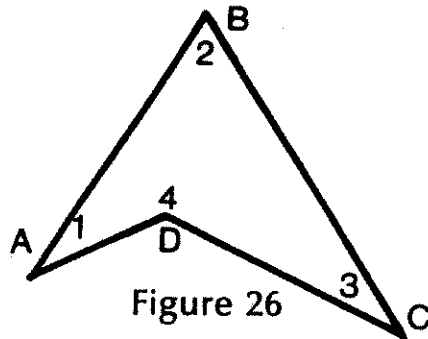


Figure 26

Follow the procedure described in Activity 26 and tessellate a plain sheet of paper. Keep marking the angles in each position of the quadrilateral as you proceed. Continue as shown in Figure 27.

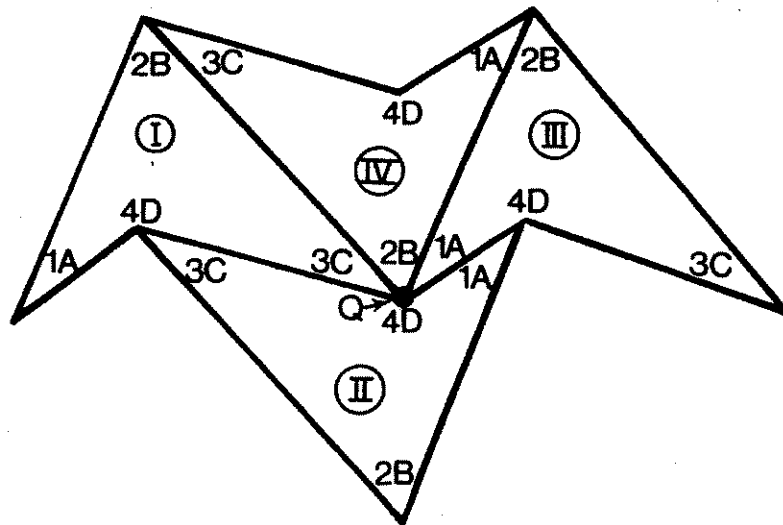


Figure 27

Look at vertex point Q in Figure 27. What is the sum of all the angles about the vertex point Q? _____

Do angles 1, 2, 3, 4 of the quadrilateral appear at each vertex point in your tessellation? Yes _____ No _____

Summary. From Activities 21–27 it should be clear that is possible to tessellate a plane with any quadrilateral.

TESSELLATION PROJECT

1. Start with a shape that you know will tessellate.
2. Modify the shape you have chosen in step #1 using either a parallel cut-out, turning on a midpoint or placing a cut-out on an adjacent side.
3. Modify the shape you have chosen in step #2 using either a parallel cut-out, turning on a midpoint or placing a cut-out on an adjacent side.
4. Tessellate on side of an 8 ½" by 11" sheet of paper with your shape. This usually works best if you make a pattern out of thin cardboard or heavy paper and trace around the pattern. Drawing shapes freehand or cutting them out of construction paper is harder and usually does not look as neat.
5. **Add coloring and other detail to make it interesting. Be creative.**
6. On a separate sheet, show how you did it. Specifically, show the results of steps 1 through 4. Add any comments or explanations you feel are appropriate. Attach your pattern piece.
7. Try to have fun with this project. Doing a tessellation can be really time consuming, so don't wait until the last minute to start it.

Due on _____, 5% grade reduction per class meeting for late projects

RUBRIC – TESSELLATION PROJECT

___ / 1	Grading rubric attached
___ / 4	Successfully doing the first cut out
___ / 4	Successfully doing the second cut out
___ / 6	Your explanation page (step #6)
___ / 3	Neatness
___ / 6	Creativity, decoration & how much I like it
___ / 24	Total