Learning Exercises for Section 2.1

3. Numbers can be expressed in a fascinating variety of ways. Different languages, of course, use different words and different symbols to represent numbers. Some counting words are given below.

English	*Spanish*	*German*	*French*	*Japanese*	*Swahili*
zero	cero	null	zero	zero	sifuri
one	uno	eins	un	ichi	moja
two	dos	zwei	deux	ni	mbili
three	tres	drei	trois	san	tatu
four	cuatro	vier	quatre	shi	nne
five	cinco	fünf	cinq	go	tano
six	seis	sechs	six	roku	sita
seven	siete	sieben	sept	shichi	saba
eight	ocho	acht	huit	hachi	nane
nine	nueve	neun	neuf	kyu	tisa
ten	diez	zehn	dix	ju	kumi

Which two sets of these counting words most resemble one another?

Why do you think that is true? Do you know these numbers in yet another language?

Spanish & French are closest
Both are european languages
English & German also have some
similarity to them, but Japanese
& Swahili are very different

4. Roman numerals have survived to a degree, as in motion picture film credits and on cornerstones. Here are the basic symbols: I = one, V = five, X = ten, L = fifty, C = one hundred, D = five hundred, and M = one thousand. For example, CLXI is 100 + 50 + 10 + 1 = 161. What numbers does each of these represent?

a. MMCXIII b. CLXXXV c. MDVII

a. $1000 + 1000 + 100 + 10 + 3 = 2113$
b. $100 + 50 + 10 + 10 + 10 + 5 = 185$
c. $1000 + 500 + 5 + 2 = 1507$

6. Other systems we have seen all involve addition of the values of the symbols. Roman numerals use a **subtractive** principle as well; when a symbol for a smaller value comes before the symbol for a larger value, the former value is subtracted from the latter. For example, IV means 5 − 1 = 4, or four; XC means 100 − 10 = 90; and CD = 500 − 100 = 400. Note that no symbol appears more than three times together, because with four symbols we would use this subtractive property. What number does each of these represent?

a. CMIII b. XLIX c. CDIX

a. (1000 − 100) + 3 = 903
b. (50 − 10) + (10 − 1) = 49
c. (500 − 100) + (10 − 1) = 409

7. Even within the same language, there are often several words for a given number idea. For example, both "two shoes" and "a pair of shoes," refer to the same quantity. What are some other words for the idea of two-ness?

twin, couple, double, duo, twice
prefix of <u>bi</u> like bicycle
prefix of <u>di</u> like di polar
see book for more

Learning Exercises for Section 2.2

1. a. How many tens are in 357? How many whole tens? 35.7, 35
   b. How many hundreds are in 4362? How many whole hundreds? 43.62, 43
   c. How many tens are in 4362? How many whole tens? 436.2, 436
   d. How many thousands are in 456,654? How many whole thousands? 456.654, 456
   e. How many hundreds are in 456,654? How many whole hundreds? 4566.54, 4566
   f. How many tens are in 456,654? How many whole tens? 45665.4, 45665
   g. How many tenths are in 23.47? How many whole tenths? 234.7, 234
   h. How many thousandths are in 23.47? How many whole thousandths? 23470
   i. How many ones are in 23.47? How many whole ones? 23.47, 23
   j. How many hundredths are in 23.47? How many whole hundredths? 2347
   k. How many tenths are in 2347? How many whole tenths? 23470, 23470
   l. How many tenths are in 234.7? How many whole tenths? 2347

3. a. Is the statement "For a set of whole numbers, the longest numeral will belong to the largest number" true or false? Why? True − more digits is large place vaue
   b. Is the statement "For a set of decimals, the longest numeral will belong to the largest number" true or false? Why?

Not necessarily − the placement of decimal point determines place value

5. Write in words the way you would pronounce each:

a. 407.053 b. 30.04 c. 0.34 d. 200.067 e. 0.276

a) Four hundred seven and fifty three thousandths

b) Thirty and four hundredths

c) Thirty Four hundredths

d) Two hundred and sixty seven thousandths

e) Two hundred Seventy six thousandths

9. Do you change the value of a whole number by placing zeros to the right of the number? To the left of the number?

2 → 20 → 200

2 → 2.0 → 2.00

If you place zeros to the right of the number you change its place value (as long as it is not to the right of the decimal place)

2 → 02 → 002

Placing zeros on the left will not affect the place value of the number

Learning Exercises for Section 2.3

2. Write ten (this many: ●●●●●●●●●●) in each given system.

a. base four 22 b. base five 20 c. base eight 12

3. Write each of these.

a. four in base four 10

b. eight in base eight 10

c. twenty in base twenty 10

d. b in base b 10

e. b^2 in base b 10

f. $b^3 + b^2$ in base b 110

g. 29_{ten} in base three

3^3	3^2	3^1	3^0
1	0	0	2

h. 115_{ten} in base five

5^2	5^1	5^0
4	3	0

i. 69_{ten} in base two

$69 - 64 = 5$, $5 - 4 = 1$

2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	0	1

j. 1728_{ten} in base twelve

12^3	12^2	12^1	12^0
1	0	0	0

4. Write the numerals for counting in base two, from one through twenty.

1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, 10010, 10011, 10100 / 10101, 10110, 10111, 11000, etc

5. How do you know that there is an error in each statement?
   a. ten = 24_{three} Base 3 only has digits 0, 1, 2, 3
   b. fifty-six = 107_{seven} Base 7 only has digits 0,1,2,3,4,5,6
   c. thirteen and three-fourths = 25.3_{four} Base 4 cannot have a 5

6. Write each of these as a base ten numeral with the usual base ten words. For example, $111_{two} = (1 \times 2^2) + (1 \times 2) + (1 \times 1) = 7_{ten}$ and $31.2_{four} = (3 \times 4) + (1 \times 1) + \frac{2}{4} = 12 + 1 + \frac{5}{10} = 13.5$, or thirteen and five-tenths.

   a. 37_{twelve} b. 37_{nine} c. 207.0024_{ten}
   d. 1000_{two} e. $1,000,000_{two}$ f. 221.2_{three}

a) $(3 \times 12) + (7 \times 1) = 36 + 7 = 43_{10}$ = fourty three
b) $(3 \times 9) + (7 \times 1) = 27 + 7 = 34_{10}$ = thirty four
c) two hundred seven and 24 ten thousandths
d) $(1 \times 2^3) = 8_{10}$ = eight
e) $(1 \times 2^6) = 64$ = sixty four
f) $(2 \times 3^2) + (2 \times 3) + (1 \times 1) + (2 \times \frac{1}{3}) =$
$18 + 6 + 1 + 2/3 = 25\ 2/3$

7. For a given number, which base—two or twelve—will usually have a numeral with more digits? What are the exceptions?

The base 2 representation will have more digits, unless it is the number 0 or 1 which are the same representation in all bases

8. In what bases would 4025_b be a legitimate numeral? if b is base 6 or larger

9. Compare these pairs of numbers by placing < or > or = in each box.
   a. 34_{five} $\boxed{<}$ 34_{six} b. 4_{five} $\boxed{=}$ 4_{six} c. 43_{five} $\boxed{>}$ 25_{six}
   d. 100_{five} $\boxed{>}$ 18_{nine} e. 111_{two} $\boxed{=}$ 7_{ten} f. 23_{six} $\boxed{>}$ 23_{five}

a) $34_{five} = 3(5) + 4 = 19_{10}$ $34_{six} = 3(6) + 4 = 22_{10}$
b) $4_{five} = 4_{10}$ $4_{six} = 4_{10}$
c) $43_{five} = 4(5) + 3 = 23_{10}$ $25_{six} = 2(6) + 5 = 17_{10}$
d) $100_{five} = 1(5^2) = 25_{10}$ $18_{nine} = 1(9) + 8 = 17_{10}$
e) $111_{two} = 1(2^2) + 1(2) + 1 = 7_{10}$
f) larger base, same number

10. On one of your space voyages, you uncover an alien document in which some "one, two, . . ." counting is done: obi, fin, mus, obi na, obi obi, obi fin, obi mus. What base does this alien civilization apparently use? Continue counting through twenty in that system.

Base 4 system, 4 numbers
na = 0, obi = 1, fin = 2, mus = 3
obi, fin, mus, obi na, obi obi, obi fin, obi mus
fin na, fin obi, fin fin, fin mus, mus na, mus obi,
mus fin, mus mus, obi na na, obi na obi,
obi na fin, obi na mus, obi obi na, etc

☆ **13.** In each number, write the "basimal" place values and then the usual base ten fraction or mixed number.

Example: $10.111_{two} = (2 + 0 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8})_{ten} = 2\frac{7}{8}$ (Recall: $4 = 2^2$ and $8 = 2^3$.)

a. 21.23_{four} **b.** 34.3_{twelve}

a) $2(4) + 1(1) + 2(\frac{1}{4}) + 3(\frac{1}{16}) = 9 + \frac{2}{4} + \frac{3}{16} = 9\frac{11}{16}$
b) $3(12) + 4(1) + 3\frac{1}{12} = 40\frac{3}{12} = 40\frac{1}{4}$

14. Write each of these in "basimal" notation.

Example: three-fourths in base ten is *what* in base two?

$$(\tfrac{3}{4})_{ten} = (\tfrac{1}{2} + \tfrac{1}{4})_{ten} = 0.11_{two}$$

a. one-fourth, in base twelve
b. three-fourths, in base twelve
c. one-fourth, in base eight

a) $\frac{1}{4} = \frac{3}{12} = 0.3_{12}$
b) $\frac{3}{4} = \frac{9}{12} = 0.9_{12}$
c) $\frac{1}{4} = \frac{2}{8} = 0.2_8$

17. Write 100_{ten} in each given base.

a. seven **b.** five **c.** eleven **d.** two **e.** thirty-one

a)

7^2	7^1	7^0
2	0	2

100 − 49 = 51, 51 − 49 = 2

b)

5^2	5^1	5^0
4	0	0

100 − 100 = 0

c)

11^2	11^1	11^0
0	9	1

100 − 99 = 1

d)

2^6 (64)	2^5 (32)	2^4 (16)	2^3	2^2	2^1	2^0
1	1	0	0	1	0	0

100 − 64 = 36, 36 − 32 = 4

e)

31^1	31^0
3	7

100 − 93

18. Complete with the proper digits.

a. $57_{ten} =$ ______ five **b.** $86_{nine} =$ ______ ten

c. $312_{four} =$ ______ ten **d.** $237_{ten} =$ ______ eight

e. $2101_{three} =$ ______ ten **f.** $0.111_{two} =$ ______ ten

a)

5^2	5^1	5^0
2	1	2

$57 - 50 = 7 - 5 = 2$

b)

9^1	9^0
8	6

$= 8(9) + 6(1) = 72 + 6 = 78$

c)

4^2	4^1	4^0
3	1	2

$= 3(16) + 1(4) + 2(1) = 54$

d)

8^2	8^1	8^0
3	5	5

$237 - 192 = 45 - 40 = 5$

e)

3^3	3^2	3^1	3^0
2	1	0	1

$= 2(27) + 1(9) + 0(3) + 1(1)$
$= 64$

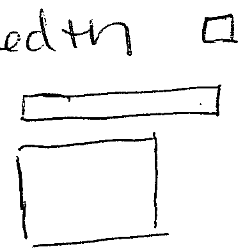
f)

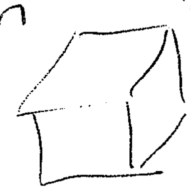
$1/2$	$1/2^2$	$1/2^3$
1	1	1

$= 1\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right) + 1\left(\frac{1}{8}\right)$
$= \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

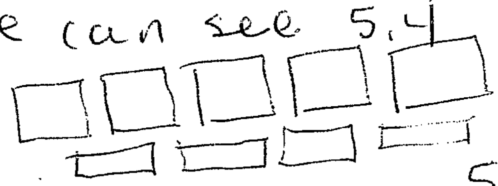
24. Represent 5.4 and 5.21 with base ten blocks, using the same block as the unit. (What will you use to represent one?) Many school children say that 5.21 is larger than 5.4 because 21 is larger than 4. How would you try to correct this error using base ten blocks?

Smallest unit is hundredths - so
let small cube = 1 hundredth
let long unit = 1 tenth
let square flat = 1 one
let large cube = 1 ten





So we can see 5.4



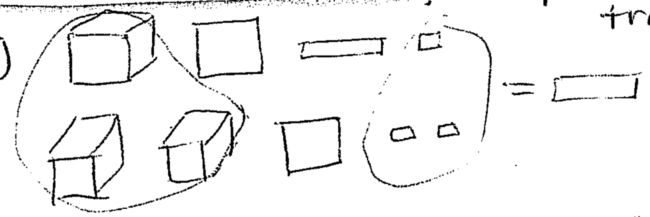
is larger than 5.21

Learning Exercises for Section 2.4

1. Add 1111_{three} and 2102_{three} without drawings and then with drawings in the ways illustrated above. Which way did you find it easier? group of 3 trade up

a) 1111_{three}
2102_{three}
10220_{three}

b)



= next unit larger $\underline{1}\ \underline{0}\ \underline{2}\ \underline{2}\ \underline{0}_{three}$

2. Do these exercises in the designated bases, using the cardboard cutouts in an appendix, or with drawings.

a. $341_{five} + 220_{five}$ = 1011_{five}

b. $101_{two} + 110_{two}$ = 1011_{two}

c. $321_{four} - 123_{four}$ = 132_{four}

d. $296_{ten} - 28_{ten}$ = 268_{ten}

4. Add the following in the appropriate bases, without blocks unless you need them.

a. $2431_{five} + 223_{five}$ = 3204_{five}

b. $351_{nine} + 250_{nine}$ = 611_{nine}

c. $643_{seven} + 134_{seven}$ = 1110_{seven}

d. $99_{eleven} + 88_{eleven}$ = 176_{eleven}

5. Subtract in different bases, without blocks unless you need them.

a. $351_{nine} - 250_{nine}$ = 101_{nine}

b. $643_{seven} - 134_{seven}$ = 506_{seven}

c. $2431_{five} - 223_{five}$ = 2203_{five}

d. $772_{eleven} - 249_{eleven}$ = 524_{eleven}

8. Use the cut-outs from an appendix for the different bases to act out the following. As you act each out, record what would take place in the corresponding numerical work.

a. $200_{four} - 13_{four}$ = 121_{four}

b. $200_{five} - 13_{five}$ = 132_{five}

c. $200_{eight} - 13_{eight}$ = 165_{eight}

d. $100_{two} - 11_{two}$ = 1_{two}