Log 1

POWERS OF TWO

Write down the first ten powers of two on one line starting with 2^0 and ending with 2^9. Write down the difference between the second number and the first number, the difference between the third number and the second number, the difference between the fourth number and the third number, and so on. What do you notice? Can you explain mathematically? (Hint: look at $2^{n+1} - 2^n$ (why?)).

Now, write down the first ten powers of two on one line again. Write the difference between the third number and the first number below the second number. Write the difference between the fourth number and the second number below the third number, the difference between the fifth number and the third number below the fourth number, and so on. What do you notice? Can you explain? (Hint: look at $2^{n+1} - 2^{n-1}$ (why?)).

Write the first ten powers of two one last time. Multiply the first and third numbers, and write the answer below the second number. Multiply the second and fourth numbers and write the answer below the third number, and so on. What do you notice? No need for proof here.

log 2

INTERNATIONAL SUMMIT

At a recent international summit, five delegates (A, B, C, D, E) participated. This is what we observed:

1. B and C spoke English, although when D joined them, they all changed to Spanish, the only common language among the three of them.

2. The only common language among A, B, and E was French.

3. The only common language between C and E was Italian.

4. Three delegates could speak Portuguese.

5. The most common language was Spanish.

6. One of the delegates spoke all five languages, another one spoke four, one spoke three, one spoke two, and the other only spoke one language.

What languages did each delegate speak?

Log 3

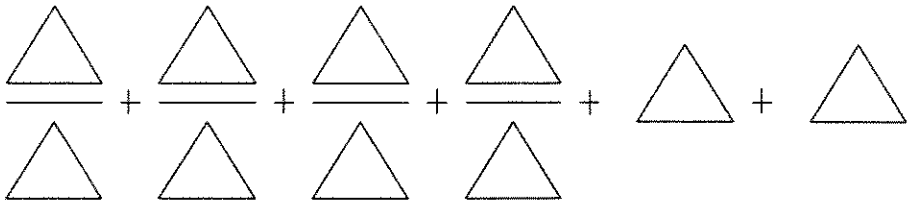
THE LOCKER PROBLEM

In a high school, there is a row of 100 lockers numbered from 1 to 100. They are all closed. 100 students line up at the beginning of the row. The first student goes down the row and opens every locker. The second student goes down the row and closes every locker that is a multiple of 2. The third student goes down the row and opens or closes every multiple of three locker (if it is open he closes it, and if it is closed he opens it). The fourth student opens or closes every multiple of four, and so on, until the 100^{th} student opens or closes the 100^{th} locker.

Which lockers are open after the 100^{th} student is done? What is the set of these numbers called? What lockers would be open if there were 200 lockers? 300? n lockers? Give a mathematical explanation for why these are the lockers that stay open.

Log 4

Fraction Fun

Consider the sum 

1) Put the digits 0, 1, 2, …, 9 in the ten triangles in such a way that the sum = 20. Use each digit only once. Explain how you approached this problem.

2. Find the largest possible sum using the digits 0 – 9 exactly once each, and explain why it is the largest.

3. Find the smallest possible sum and explain why it is the smallest.

Log 5

MAILMAN MARTY

Mailman Marty delivers the mail in the small village of Tenhouses. This village, as you already suspected, has only one street with exactly ten houses, numbered from 1 up to and including 10.

In a certain week, Marty did not deliver any mail to two houses in the village; at the other houses he delivered mail exactly three times each. Each working day he delivered mail to exactly four houses.

The sums of the house numbers where he delivered mail were:
on Monday: 18
on Tuesday: 12
on Wednesday: 23
on Thursday: 19
on Friday: 32
on Saturday: 25
on Sunday: he never works.

Which two houses did not get any mail this particular week?

+5 pts. extra credit if you determine which houses got mail on which days