14.4 /15

DIFFERENCE PATTERNS

Complete each table. Write the rule and find the difference of successive y values.

Example: Rule: $y = (3x - 2)$

x	y	D_1
1	1	
2	4	3 (4 – 1)
3	7	3 (7 – 4)
4	10	3 (10 – 7)
5	13	3 (13 – 10)

1. Rule: $y =$ _____

x	y	D_1
1	5	
2	7	2
3	9	___
4	___	___
5	___	___

2. Rule: $y =$ _____

x	y	D_1
1	3	
2	7	___
3	11	___
4	___	___
5	___	___

3. Rule: $y =$ _____

x	y	D_1
1	7	
2	12	___
3	17	___
4	___	___
5	___	___

Now find D_1 (difference of y's) and D_2 (difference of D_1) for these quadratic relations.

Example: Rule: $y = x^2 + 1$

x	y	D_1	D_2
1	2		
2	5	3	
3	10	5	2
4	17	7	2
5	26	9	2

4. Rule: $y =$ _____

x	y	D_1	D_2
1	4		
2	10	___	
3	20	___	___
4	34	___	___
5	___	___	___

5. Rule: $y =$ _____

x	y	D_1	D_2
1	2		
2	11	___	
3	26	___	___
4	47	___	___
5	___	___	___

6. Rule: $y =$ _____

x	y	D_1	D_2
1	1		
2	13	___	
3	33	___	___
4	61	___	___
5	___	___	___

Find My rule

in	out
1	4
2	7
3	10
4	13
5	16
n	

in	out
3	4
4	2
5	0
6	-2
7	-4
n	

in	out
1	3
2	9
3	19
4	33
5	51
n	

in	out
2	-9
3	-20
4	-35
5	-54
6	-77
n	

DIFFERENCE PATTERNS

Key

Complete each table. Write the rule and find the difference of successive y values.

Example: Rule: $y = (3x - 2)$ Rule: $y =$ $2x+3$ Rule: $y =$ $4x-1$ Rule: $y =$ $5x+2$

x	y	D_1
1	1	
2	4	3 (4 − 1)
3	7	3 (7 − 4)
4	10	3 (10 − 7)
5	13	3 (13 − 10)

1.

x	y	D_1
1	5	
2	7	2
3	9	2
4	11	2
5	13	2

2.

x	y	D_1
1	3	
2	7	4
3	11	4
4	15	4
5	19	4

3.

x	y	D_1
1	7	
2	12	5
3	17	5
4	22	5
5	27	5

Now find D_1 (difference of y's) and D_2 (difference of D_1) for these quadratic relations.

Example: Rule: $y = x^2 + 1$ Rule: $y =$ $2x^2+2$ Rule: $y =$ $3x^2-1$ Rule: $y =$ $4x^2-3$

x	y	D_1	D_2
1	2		
2	5	3	
3	10	5	2
4	17	7	2
5	26	9	2

4.

x	y	D_1	D_2
1	4		
2	10	6	
3	20	10	4
4	34	14	4
5	52	18	4

$\frac{1}{2}(4) = 2$

5.

x	y	D_1	D_2
1	2		
2	11	9	
3	26	15	6
4	47	21	6
5	74	27	6

$\frac{1}{2}(6) = 3$

6.

x	y	D_1	D_2
1	1		
2	13	12	
3	33	20	8
4	61	28	8
5	97	36	8

$\frac{1}{2}(8) = 4$

Find My rule

in	out
1	4
2	7
3	10
4	13
5	16
n	

$3n+1$

in	out
3	4
4	2
5	0
6	-2
7	-4
n	

$-2n+10$

in	out
1	3
2	9
3	19
4	33
5	51
n	

$2n^2+1$

in	out
2	-9
3	-20
4	-35
5	-54
6	-77
n	

$-2n^2-n+1$

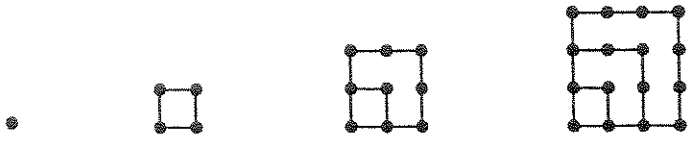
COUNTING DOT PATTERNS

The first four terms of some patterns of dots are shown below. Look for number patterns that would give a rule to help find the fifth, tenth, and *n*th terms in the sequence.

1. a. How many dots in the fifth term? ____

b. How many dots in the tenth term? _____

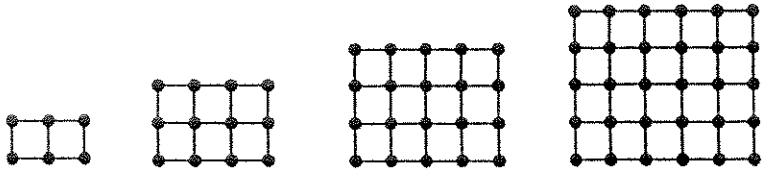
c. How many dots in the *n*th term? _____



2. a. How many dots in the fifth term? ____

b. How many dots in the tenth term? _____

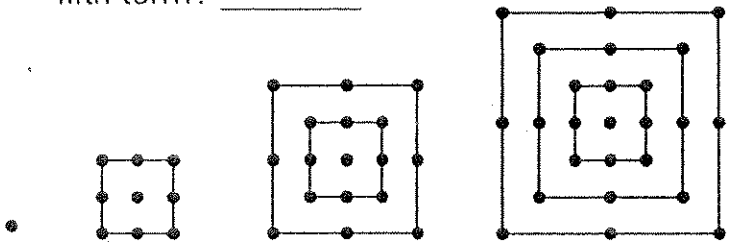
c. How many dots in the *n*th term? _____



3. a. How many dots in the fifth term? _____

b. How many dots in the tenth term? _____

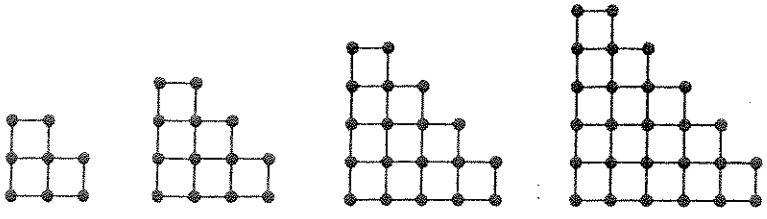
c. How many dots in the *n*th term? _____



4. a. How many dots in the fifth term? ____

b. How many dots in the tenth term? _____

c. How many dots in the *n*th term? _____



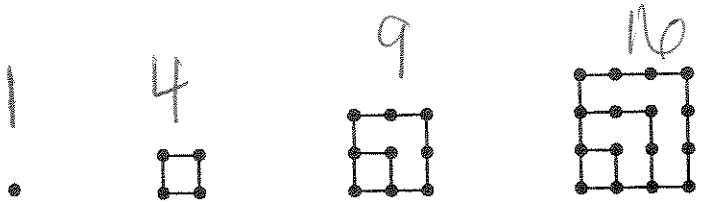
COUNTING DOT PATTERNS

The first four terms of some patterns of dots are shown below. Look for number patterns that would give a rule to help find the fifth, tenth, and *n*th terms in the sequence.

1. a. How many dots in the fifth term? 25

b. How many dots in the tenth term? 100

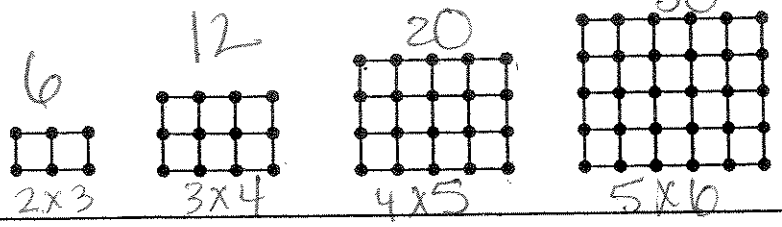
c. How many dots in the *n*th term? n^2



2. a. How many dots in the fifth term? $6 \times 7 = 42$ $(n+1)(n+2)$

b. How many dots in the tenth term? $11 \times 12 = 132$

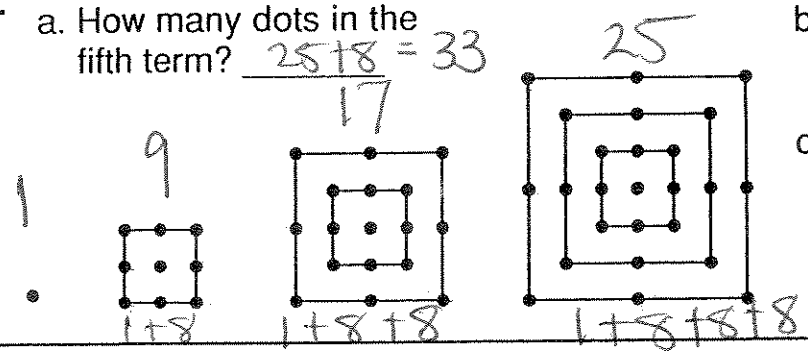
c. How many dots in the *n*th term? $(n+1)(n+2)$



3. a. How many dots in the fifth term? $25 + 8 = 33$

b. How many dots in the tenth term?

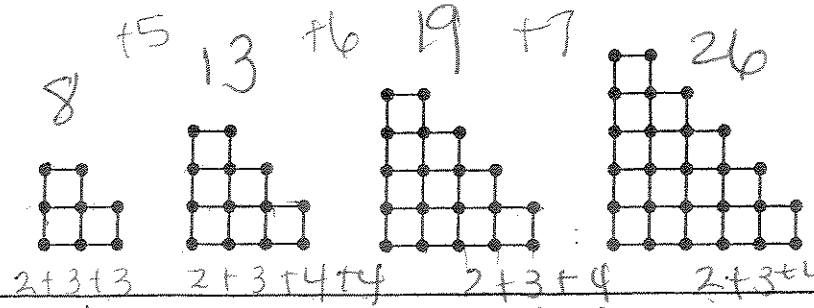
c. How many dots in the *n*th term? $(n+1)(8)+1$



4. a. How many dots in the fifth term? 34

b. How many dots in the tenth term? 178

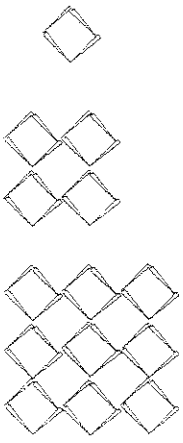
c. How many dots in the *n*th term? n^2+7n+8



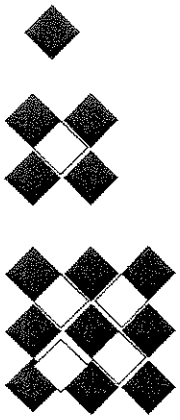
Function Patterns:

Find the general rule for each of the following:

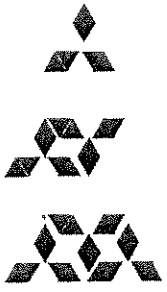
The number of toothpicks



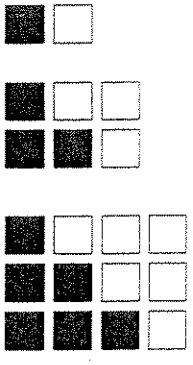
The number of squares



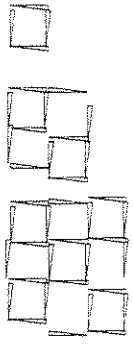
Number of rhombi



Number of squares



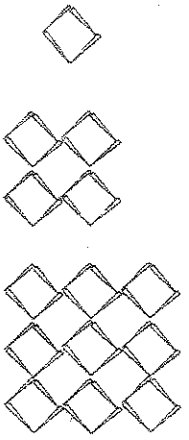
Number of toothpicks



Function Patterns;

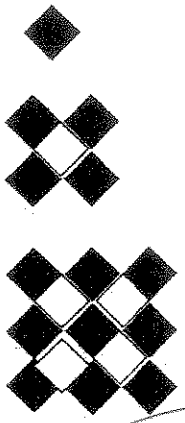
Find the general rule for each of the following:

The number of toothpicks



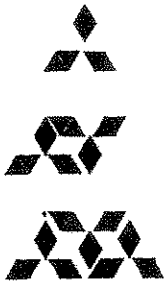
$(2n)^2$

The number of squares



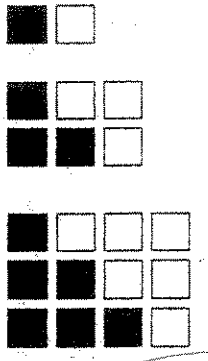
n^2

Number of rhombi



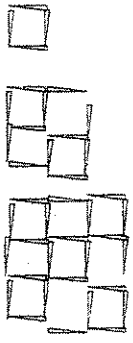
$3n$

Number of squares



$\frac{n(n+1)}{2}$

Number of toothpicks



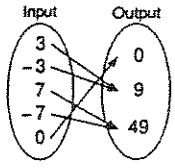
$2n^2+2n$

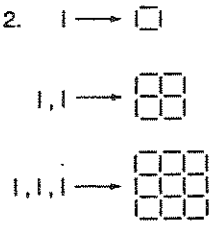
Familiar Functions

Mathematicians have always been interested in relationships between sets of numbers. These relationships have been studied for thousands of years. Some of these relationships are too complex for nonmathematicians to understand. However, many other relationships are interesting and simple enough for us to study.

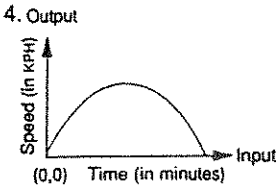
In mathematics class, Mrs. Perez introduced a new term, *function*. She said, "Some of the examples shown are special types of relations that can be called *functions*." Then she added, "To understand functions, you also need to see examples of relations that are *not* functions. Some of the examples shown are *not* functions."

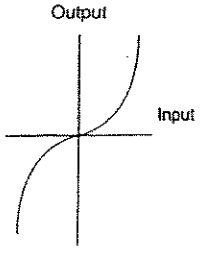
Functions

1. 

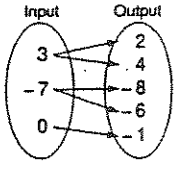
2. 

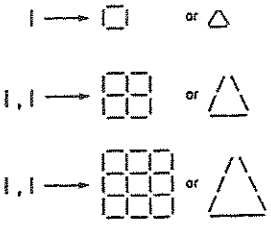
3. {(1, 2), (7, 8), (16, 17), (−3, − 2)}

4. 

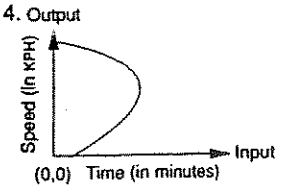
5. 

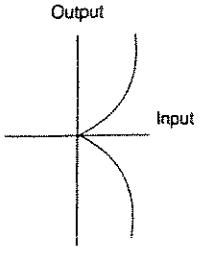
Not Functions

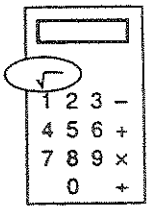
1. 

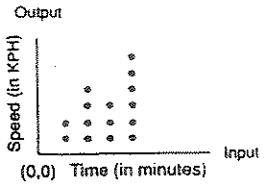
2. 

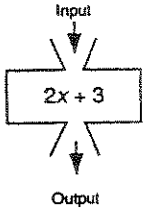
3. {(1, 2), (1, 8), (1, 1 7), (1, − 2)}

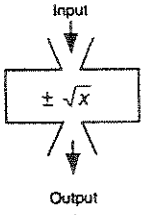
4. 

5. 

6. 

6. 

7. 

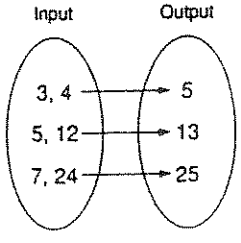
7. 

What do you think is the difference between relations that are functions and those that are not? _____

If you have not learned the definition of a function, the statement you made is a *conjecture*. Your conjecture states what you think is the definition of a function on the basis of the examples you were given. Check to see that your *conjecture* works to explain *all* the examples given.

1. Guy and Oma are trying to figure out which of the following examples are functions. Use your conjecture to help them decide by circling yes or no for each example.

a) Yes No



b) Yes No

c) Yes No

{(2, 4), (4, 2), (12, 13), (13, 12)}

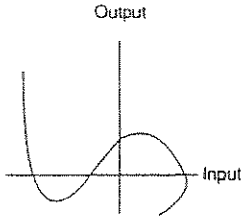
d) Yes No

{(−1, 2), (1, 2), (−3, 8), (−3, 7)}

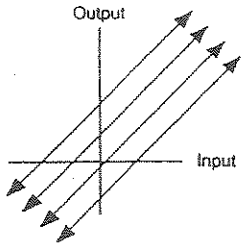
The editors wish to thank Ron Vavrinek and Susan K. Eddins, Illinois Mathematics and Science Academy, Aurora, IL 60506-1039, for writing this issue of the *NCTM Student Math Notes*.

Familiar Functions—Continued

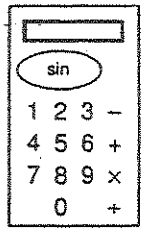
e) Yes No



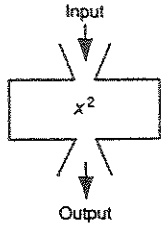
f) Yes No



g) Yes No



h) Yes No



2. Explain the reasoning you used for the examples in problem 1. _____

Mrs. Perez told the students that to understand a function you must understand two things: (1) what it can accept as an input and (2) what *unique* output it gives for each input it can accept. She gave them this example:

Input	Output
$\frac{1}{3}$	$0.\overline{3}$
$\frac{3}{4}$	0.75
$\frac{1}{5}$	0.2
$\frac{2}{9}$	$0.\overline{2}$
$\frac{1}{11}$	$0.\overline{09}$

3. What inputs does it appear that this function can accept? _____

4. How do you think the function gets an output from each input? _____

5. What outputs would the function give for inputs of 1/6?_____ 4/7?_____ 1/8?_____ 9/10?_____ 1/30?_____

6. What input would the function need to give an output of 0.0625? _____

Some functions use more than one input to compute each output.

Input	(5, 3)	(10, 4)	(6, 0)	(7, 7)	(2, 9)	$\left(\frac{5}{7}, \frac{3}{7}\right)$	(−5, 4)
Output	2	6	6	0	−7	$\frac{2}{7}$	−9

7. Describe the inputs it appears that this function can accept. _____

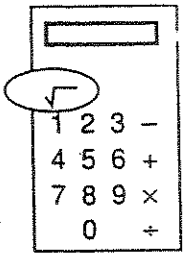
8. Describe the "rule" that you think the function uses to get an output from each input. _____

9. Use this rule to complete the following table using these inputs.

Input	(98, 16)	$\left(\frac{5}{8}, \frac{1}{4}\right)$	(23, 23)	(3, 17)	(0.32, 0.478)	(−4, 7)
Output						

10. List three input pairs for which the function would give an output of 3. _____

Calculators as Function Machines



11. Give the calculator's output when the $\sqrt{\ }$ function key is applied to each of these inputs. 25_____, 45796 _____, 318 _____, 0.25 _____, 0.37 _____, 0 _____, −16 _____, −523_____, −0.861 _____

12. For what type of input was the calculator not able to give a numerical output? _____

13. Complete the table for the function x^y. (A scientific calculator may be needed for some entries.) (Using parentheses around negative numbers is a good idea with calculators.)

Input x	3	2	1	0	4	0	2	−4	36	32	−36	−32
Input y	2	3	17	4	0	−4	−3	0	$\frac{1}{2}$ or 0.5	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
Output x^y		8		0	1							

14. For what type of input pairs was the calculator *not* able to give a numerical output? _____

15. If you wanted to test your conjecture from problem 14, what pairs of numbers would you input? _____

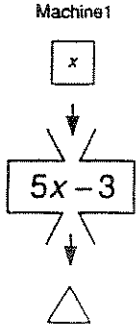
16. For the same x^y function, complete the following table. (Multiple answers may be possible.)

Input x	2				1	1
Input y		3	2			
Output x^y	16	27	9	64	1	3

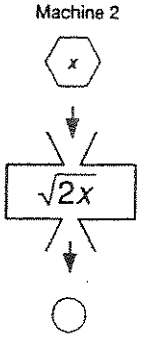
Function Machines

Functions can be defined through using function machines like those in problem 17. The input for machine 1 is whatever is in the square; for machine 2 it is whatever is in the hexagon. The output from machine 1 appears in the triangle; for machine 2, the output appears in the circle.

17. Complete the tables.

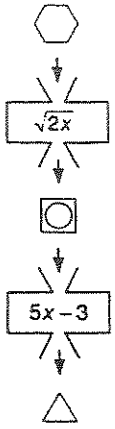


Machine 1	
Square	Triangle
−5	
	0
10	
0	
	−7
12	



Machine 2	
Hexagon	Circle
8	
	5
	0
−7	
	6
	−2

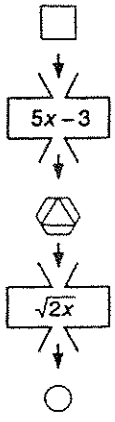
Linking Function Machines



Machine 1 and machine 2 have been linked so that the output of machine 2 becomes the input for machine 1.

18. Complete this table.

Hexagon	Machine 2 output = machine 1 input	Triangle
8		
5		
0		
−7		
18		



Next, machine 1 and machine 2 are hooked together in a different way. This time the output of machine 1 becomes the input for machine 2.

19. Complete this table.

Square	Machine 1 output = machine 2 input	Circle
−5		
0.6		
10		
0		
−0.8		
12		

MATH 313 – REVIEW 14.4 & 15

Identify properties – commutative (+, x), associative (+, x), multiplicative identity, additive identity, and distributive property
Show how operations in whole numbers can inform operations with polynomials
Perform addition, subtraction, multiplication & division with polynomials
Identify and continue patterns
Identify functions
Use "function machines"
Find combinations of functions numerically and algebraically
Write an algebraic formula for the "nth" term in pattern problems involving pictures or numbers.
Find algebraic formulas for the "nth" term in arithmetic sequences
Find algebraic formulas for the "nth" term in geometric sequences
Find algebraic formulas for the "nth" term in sequences involving n^2
Find algebraic formulas for the "nth" term in other types of sequences
Find algebraic formulas for sums
Find patterns for powers of numbers
Find algebraic formulas for perimeter problems