Applications of Exponentials & Logarithmic Functions

The general exponential growth function is $P = P_0 e^{kt}$ for $k > 0$

1. Suppose that the bacteria in a lab culture increase from an initial population of 500 to 1500 in three days. The lab needs a total of 10,000 of the bacteria. How many days will it take for the population to reach a size of 10,000?

2. The population of a city triples every 15 years and the current population is 140,000. How long will it take for the current population to double?

The interest earned on an investment is calculated in several ways.

Compound interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Continuous compound interest: $A = Pe^{rt}$

3. You have \$15,000 to put in a savings account at one of two banks for 5 years. Union Bank offers 3.4% interest on their savings accounts compounded monthly. Pacific Bank offers the same interest rate compounded continuously, but they charge a yearly service fee of \$65 on all savings accounts.
   a) Which bank will leave you with the most money at the end of five years?
   b) How long will it take you to have \$17,500 in that account?
   c) How long will it take you to double your money?

Exponential decay is modeled by the growth function $P = P_0 e^{-kt}$ for $k < 0$

4. The decay of a radioactive substance can be modeled using the function above. A materials half-life is the amount of time it takes for the substance to reduce to half its original amount. When a living organism dies, it stops absorbing the carbon-14 in its body, so the carbon isotope begins to decay. Carbon-14 has a half-life of 5750 years. The remains of a prehistoric man has only 1/3 of the carbon-14 it would have had living. How old are the remains?

In an amplifier, if E_I is the input voltage and E_O is the output voltage, then the decibel gain is given by: db gain $= 20 \cdot \log\left(\frac{E_O}{E_I}\right)$

5. If the input voltage to an amp is 0.5 volts, and the output voltage is 40 volts, find the decibel gain.

6. Suppose that the amp is supposed to create a decibel gain of 60 decibels and the voltage input from a guitar is 0.5, what must the voltage output be?

The Richter Scale is a logarithmic scale. If R is the intensity of the earthquake, A is the amplitude (measured in micrometers), and P is the period (the time it takes for the earth's surface to make one oscillation), then $R = \log\left(\frac{A}{P}\right)$

7. In a recent earthquake, the earth moved 5650 micrometers from its regular position and oscillated with a period of 0.3 seconds. What would this earthquake measure on the Richter scale?

8. Trends suggest that an earthquake will come that measures 5.8 on the Richter scale. Geological and statistical research suggests that the period will be 0.3 What is a good prediction for the amplitude?

"What do you call pants worn in a flood?"

Solve each equation. If answer is not exact, round only on your last step, round answer to the nearest hundredth. The answer to each problem will match a letter that will allow you to figure out the joke.

1. $3^x = 243$	T. 4
2. $2^x = \frac{1}{16}$	R. ½
3. $5^{x+2} = 25^x$	W. 0.62
4. $8^{2x} = 16$	A. 5
5. $(2^{3x})(2^{5x}) = 16$	E. 994.65
6. $3^x = 8$	D. 1.61
7. $7^{2x} = 11$	B. 3
8. $5^x = 3^{x+1}$	L. 12
9. $e^x = 5$	S. -4
10. $e^x \cdot e^{3x} = 4$	I. 1096.63
11. $\log_7 x = 2$	C. 2.15
12. $\log_5 125 = x$	H. 2/3
13. $\log_3(2x-1) = 3$	O. 0.35
14. $\log_7(3x-11) = \log_7(x-3)$	U. 14
15. $\log_6 x + \log_6 3 = 2$	V. 49
16. $\ln x = 7$	R. 1.89
17. $\ln(x-1) + \ln 3 = 8$	E. 2
18. $4\ln x - 2\ln 2 = \ln 128$	
19. $\ln e^{2x} = 6$	

19	6	16	14	8	4	3	2	10	11	17	5

18	5	10	13	12	15	17	9	7	1	14	3	6	2