18.5

Regular Polyhedra

Preliminary ideas:

1. The sum of the angles in any polygon (n-gon) is equal to (n-2)180°.

   a. Understand why the sum of the angles in any triangle is 180°.

   b. Understand that any polygon (n-gon) can always be "cut" into (n-2) triangles, without any new vertices being created.

   c. Understand how all the angles of these triangles "form" all the angles of the polygon, and therefore the sum of the angles in any polygon is (n-2)180°.

2. Each angle in a regular polygon (n-gon) is one-n^{th} ($^1/_n$) of the sum of its angles ((n-2)180°/n).

   a. Know that all the angles are equal in a regular (or equiangular) polygon.

   b. And therefore, the sum of its angles must be divided into n equal parts to form n equal angles.

Main ideas:

3. Understand what is meant by regular polyhedra.

   a. Examine models of (convex) regular polyhedra.

   b. Identify shared characteristics between the (convex) regular polyhedra.

   c. Develop a definition for regular polyhedra. (Note: this should include the facts that they have: (1) congruent regular polygonal faces and (2) the same arrangement at each vertex).

4. Understand & justify why there are only **five** (convex) regular polyhedra.

   a. Understand that the polygon with the fewest number of sides is a triangle and therefore for polygons (n-gons) in general "n" must be greater than or equal to 3.

   b. Understand why the fewest number of faces surrounding a vertex ($^F/_V$) of a polyhedron is 3.

   c. Know why the sum of the angles surrounding a vertex of a polyhedron must be less than 360°.

   d. Develop a table of possible (convex) regular polyhedra using what you now know.

INFORMAL PROOF OF 5 REGULAR POLYHEDRA

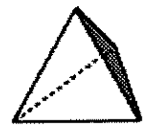
Each face of a regular polyhedra must be a regular polygon. Also the sum of the angles at each vertex must total less than 360°, and at least 3 polygons must intersect at the vertex.

Shape	Angle	How many meet at vertex	Sum of angles at vertex
triangle	60°	3	180° OK
triangle	60°	4	240° OK
triangle	60°	5	300° OK
triangle	60°	6	360° not OK
square	90°	3	270° OK
Square	90°	4	360° not OK
pentagon	108°	3	324° OK
hexagon	120°	3	360° not OK

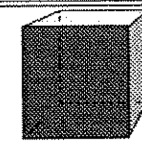
Any polygon with more sides than 6 will have angles larger than 120° and three anges coming together will form an angle larger than 360°, and will not work.

Regular Polyhedra or Platonic Solids:
Tetrahedron, Cube, Octahedron, Dodecahedron, Icosahedron

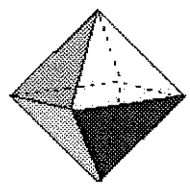
There are only five geometric solids that can be made using a regular polygon and having the same number of these polygons meet at each corner. The five Platonic solids (or regular polyhedra) are the tetrahedron, cube, octahedron, dodecahedron, and icosahedron.



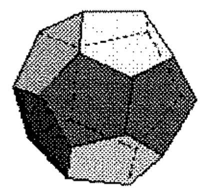
TETRAHEDRON



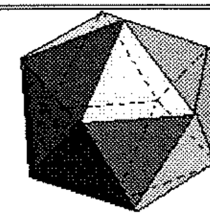
CUBE



OCTAHEDRON



DODECAHEDRON

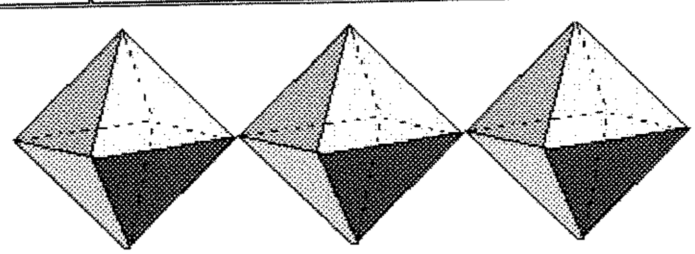


ICOSAHEDRON

The five regular polyhedra were discovered by the ancient Greeks. The Pythagoreans knew of the tetrahedron, the cube, and the dodecahedron; the mathematician Theaetetus added the octahedron and the icosahedron. These shapes are also called the Platonic solids, after the ancient Greek philosopher Plato; Plato, who greatly respected Theaetetus' work, speculated that these five solids were the shapes of the fundamental components of the physical universe.

Platonic Solids - Fact Sheet

Platonic Solid	Number of Faces	Shape of Faces	Number of Faces at Each Vertex	Number of Vertices	Number of Edges	Dual (The Platonic Solid that can be inscribed inside it by connecting the mid-points of the faces)
Tetrahedron	4	Equilateral Triangle (3-sided)	3	4	6	Tetrahedron
Cube	6	Square (4-sided)	3	8	12	Octahedron
Octahedron	8	Equilateral Triangle (3-sided)	4	6	12	Cube
Dodecahedron	12	Pentagon (5-sided)	3	20	30	Icosahedron
Icosahedron	20	Equilateral Triangle (3-sided)	5	12	30	Dodecahedron



That these five solids are the only possible regular solids (that is, all faces are congruent and all solid angles at the vertices are equal) was one of the great discoveries of the ancient Greeks. Euclid discussed them at length in his book *Elements*. Plato theorized that all matter was made

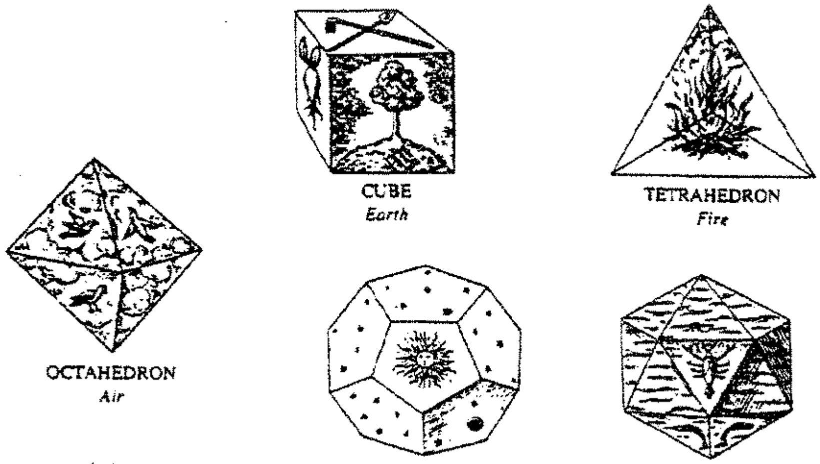


Figure 27.5 The five regular solids as depicted by Johannes Kepler in *Harmonices Mundi, Book II* (1619).

up of minute particles of earth, air, fire, and water. He believed that earth particles were cubes, air particles were octahedra, fire particles were tetrahedra, and water particles were icosahedra. Pythagoras and his followers mystically associated the dodecahedron with the cosmos. They believed that understanding of the dodecahedron was too dangerous for ordinary people and tried to restrict this knowledge to their own cult. Over 2000 years later, the astronomer Johannes Kepler tried, in vain, to model the planetary system after the way in which the five regular solids can be inscribed in one another. It was his conjecture that the spheres of the planets were circumscribed and inscribed by these solids. (See Figure 27.6.)

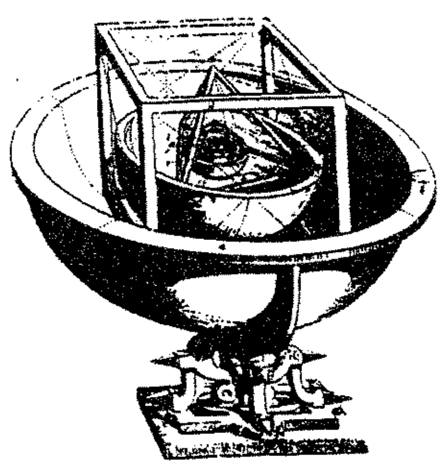


Figure 27.6 Kepler's cosmic mystery pictures the spheres of the six planets nested in the five perfect solids of Pythagoras and Plato. The outermost solid is the cube.