PASCAL'S TRIANGLE

THE SOURCE OF MANY PATTERNS

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1

SUMMING UP PASCAL

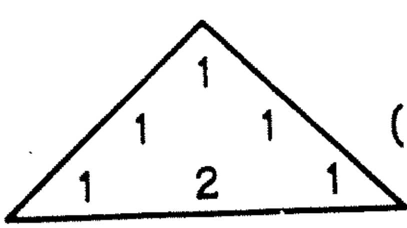
Find the sum of <u>all</u> the numbers in a Pascal triangle with 8 rows. Add the numbers in a Pascal triangle with 1 row. Do the same for a triangle with 2 rows, 3 rows, etc. until you see the pattern that emerges.

Record your results in the table shown.

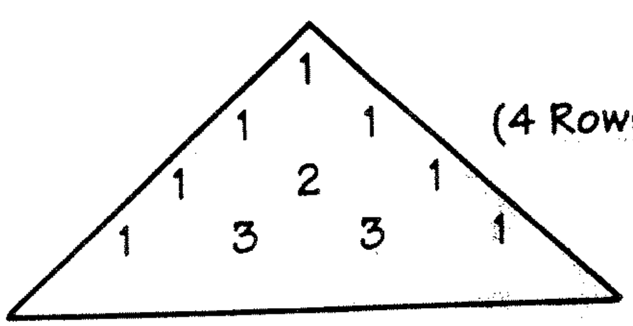
What is the sum of all the numbers in a Pascal triangle with
(a) 50 rows?
(b) n rows?

(1 Row)

(2 Rows)



(3 Rows)

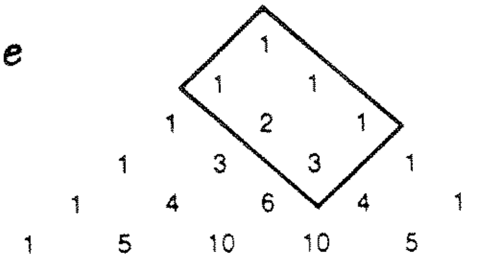


(4 Rows)

Numbers of Rows in Triangle	Sum of all Numbers in Triangle
1	1
2	3
3	7
4	
5	
6	
7	
8	
50	
n	

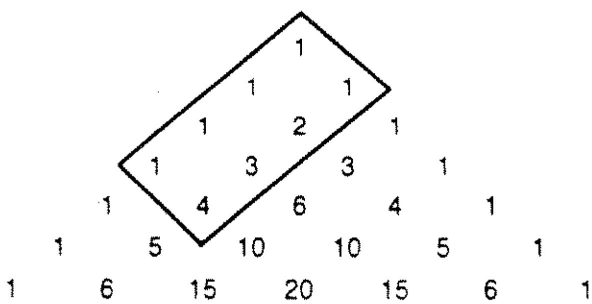
PASCAL MAGIC

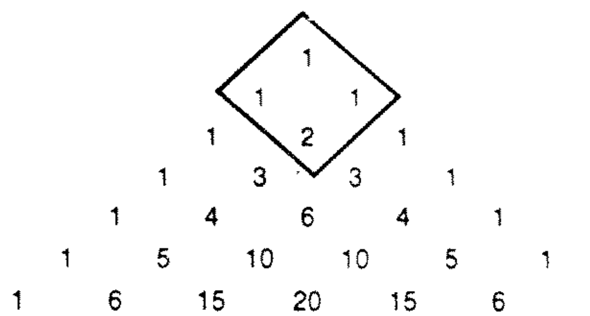
Examine Pascal's triangle to the right. Notice that the sum of all the numbers contained inside the parallelogram is 9.



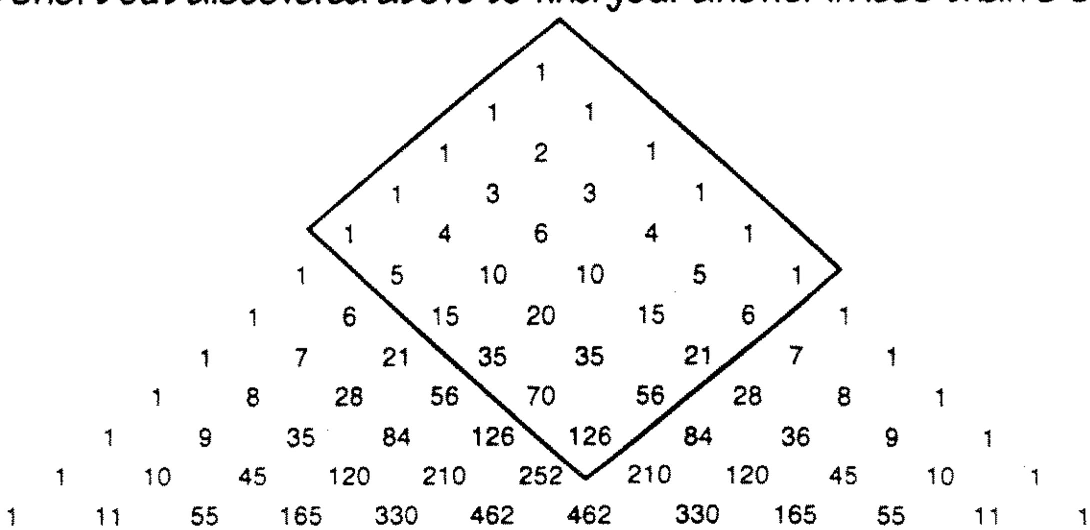
One of the numbers on the outside of the parallelogram can help you find the sum very quickly.

Find the sum of all the numbers contained in each parallelogram and relate the sum to one of the numbers found outside the parallelogram.





Find the sum of all the numbers contained in the parallelogram below.
Use the short cut discovered above to find your answer in less than 3 seconds!

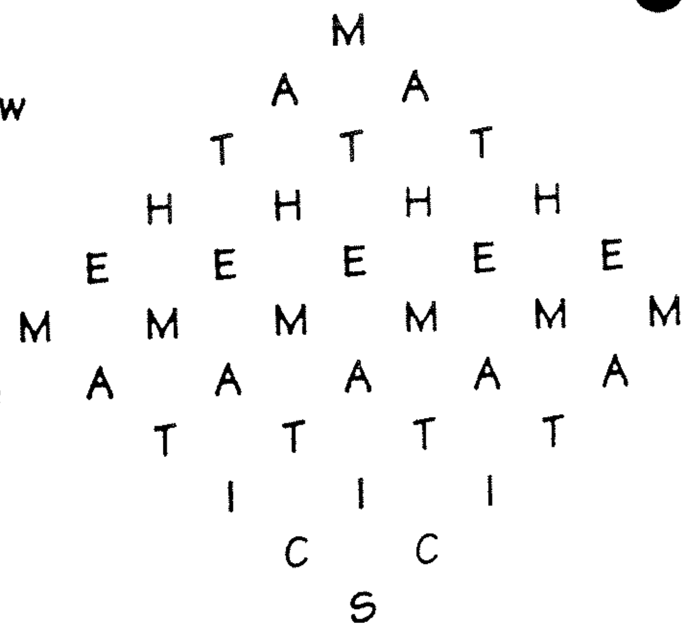


Use a calculator to check your results.

Draw your own parallelogram and find the "magic" sum.
Note: The parallelograms must always contain the top "1" on Pascal's triangle.

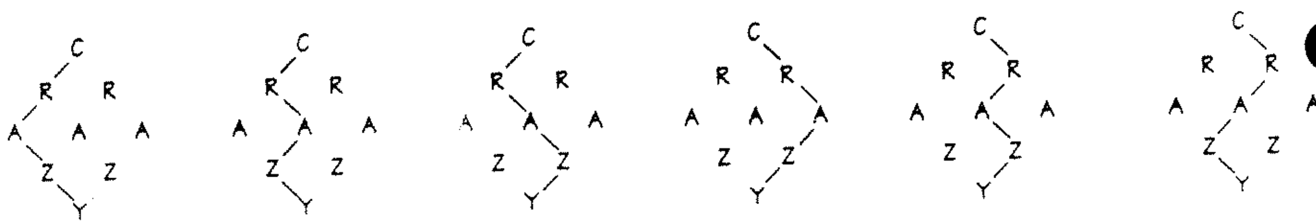
MATHEMATICS

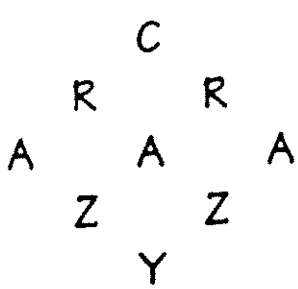
Examine the arrangement of letters to the right. Starting at the top, how many *downward paths* can you take to spell "MATHEMATICS"?



First, try an easier problem. How many paths can you take to spell "CRAZY" assuming you start at the top and move diagonally down?

There are six possible paths. They are shown below.





If you place these letters on top of Pascal's triangle, with the "C" on the top "1," the triangle will reveal the solution to this problem!

Now, using Pascal's triangle, try to solve the original problem.
Make up your own problems, using any word with an odd number of letters.
Let Pascal's triangle provide the solution!

Border Patterns

Look for a pattern to write an expression for the number of ●s in each figure.

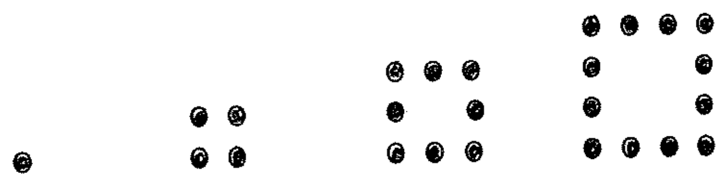


Fig 1 Fig 2 Fig 3 Fig 4 Fig 5 Fig 6

(a) Sketch the next 2 figures.

(b) Complete the following table.

figure number								50	**n**
number of ●s									

(c) Write a description of how the ●s are arranged for the 50th figure. How many ●s will there be? Include this information in the table.

(d) Write a description of how the ●s are arranged for the **n**th figure.

(e) Write a general expression for the number of ●s in the **n**th figure. Include this information in the table.

Pattern Plusses

Look for a pattern to write an expression for the number of ●s in each figure.

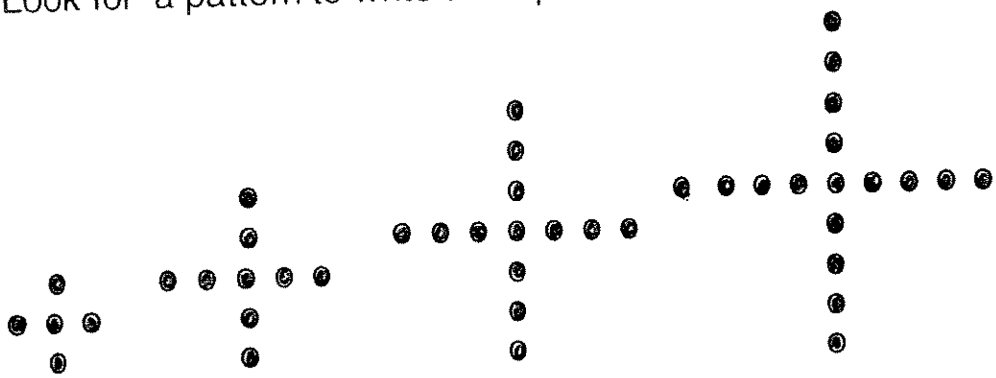


Fig 1 Fig 2 Fig 3 Fig 4

(a) Sketch the next 2 figures.

(b) Complete the following table.

figure number										n
number of ●s										

(c) Write a description of how the ●s are arranged for the 50th figure. How many ●s will there be? Include this information in the table.

(d) Write a description of how the ●s are arranged for the **n**th figure.

(e) Write a general expression for the number of ●s in the **n**th figure. Include this information in the table.

U Patterns

Look for a pattern to write an expression for the number of os in each figure.

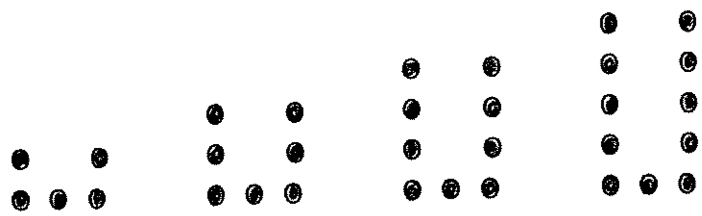


Fig 1 Fig 2 Fig 3 Fig 4 Fig 5 Fig 6

(a) Sketch the next 2 figures.

(b) Complete the following table.

figure number										n
number of ●s										

(c) Write a description of how the ●s are arranged for the 50th figure. How many ●s will there be? Include this information in the table.

(d) Write a description of how the ●s are arranged for the **n**th figure.

(e) Write a general expression for the number of ●s in the **n**th figure. Include this information in the table.

Patterns: Try Angle?

Look for a pattern to write an expression for the number of +s in each figure.

Fig 1	Fig 2	Fig 3	Fig 4	Fig 5	Fig 6
+	+ + + +	+ + + + + + + + +	+ + + + + + + + + + + + + + + +		

(a) Sketch the next 2 figures.

(b) Complete the following table.

figure number											n
number of +s											

(c) Write a general expression for the number of +s in the **n**th figure. Include this information in the table.

(d) Graph the data in the table. Label the axes.

If you connect the points will the shape of the graph be a straight line?

DIFFERENCE PATTERNS

◆◆◆

Complete each table. Write the rule and find the difference of successive y values.

Example: Rule: $y = (3x - 2)$

x	y	D_1
1	1	
2	4	3 (4 − 1)
3	7	3 (7 − 4)
4	10	3 (10 − 7)
5	13	3 (13 − 10)

1\. Rule: $y =$ ________

x	y	D_1
1	5	
2	7	2
3	9	___
4	___	___
5	___	___

2\. Rule: $y =$ ________

x	y	D_1
1	3	
2	7	___
3	11	___
4	___	___
5	___	___

3\. Rule: $y =$ ________

x	y	D_1
1	7	
2	12	___
3	17	___
4	___	___
5	___	___

Now find D_1 (difference of y's) and D_2 (difference of D_1) for these quadratic relations.

Example: Rule: $y = x^2 + 1$

x	y	D_1	D_2
1	2		
2	5	3	
3	10	5	2
4	17	7	2
5	26	9	2

4\. Rule: $y =$ ________

x	y	D_1	D_2
1	4		
2	10	___	
3	20	___	___
4	34	___	___
5	___	___	___

5\. Rule: $y =$ ________

x	y	D_1	D_2
1	2		
2	11	___	
3	26	___	___
4	47	___	___
5	___	___	___

6\. Rule: $y =$ ________

x	y	D_1	D_2
1	1		
2	13	___	
3	33	___	___
4	61	___	___
5	___	___	___

Now find D_1, D_2, and D_3.

7\. Rule: $y =$ ________

x	y	D_1	D_2	D_3
1	2			
2	9	___		
3	28	___	___	
4	65	___	___	___
5	126	___	___	___
6	___	___	___	___

8\. Rule: $y =$ ________

x	y	D_1	D_2	D_3
1	−1			
2	13	___		
3	51	___	___	
4	125	___	___	___
5	247	___	___	___
6	___	___	___	___

Use your results to complete the following statements:

9\. In a rule where the highest power of x is 1 then the first differences are ________________.

10\. In a rule where the highest power of x is 2 then the second differences are ________________.

11\. In a rule where the highest power of x is 3 then the ______________ differences are ________________.