

1 PLACE

10 PLACE

100 PLACE

1000 PLACE

ROMAN NUMERATION SYSTEM

The following list gives the symbols used by the later Romans to represent certain numbers.

<u>Amount</u>	<u>Symbol</u>
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

Find the value of each of the following numbers written using Roman numeration.

1. MXI
2. MIX (what is the difference here?)
3. DXCVII

How would the Romans represent the following amounts?

4. 1102
5. 319
6. 1254

How does the Roman numeration system work?

What similarities or differences do you notice from a simple tally mark system?

What limitations or disadvantages do you find to this system?

1. 2103_{four} has four digits. The first four place values in base four are written here, and the given digits put in there places:

$$\begin{array}{cccc} \underline{2} & \underline{1} & \underline{0} & \underline{3} \\ \text{of four sixteens,} & \text{of four fours, or} & \text{of four ones, or} & \text{ones, or } 4^0 \\ \text{or sixty-four, or } 4^3 & \text{sixteen, or } 4^2 & \text{four, or } 4^1 & \end{array}$$

2. What does the 2 tell us? The 2 stands for two of 4^3 which is $2 \times 64 = 128$ in base ten.
3. What does the 1 tell us? The 1 stands for one 4^2 which is $1 \times 16 = 16$ in base ten.
4. What does the 0 tell us? The 0 stands for zero of 4^1 which is $0 \times 4 = 0$ in base ten.
5. What does the 3 tell us? The 3 indicates 4^0 is used three times, $3 \times 1 = 3$ in base ten.
6. Thus $2103_{\text{four}} = (128 + 16 + 0 + 3)_{\text{ten}} = 147_{\text{ten}}$
that is, $2103_{\text{four}} = 147_{\text{ten}}$.

Example: Suppose instead we want to change a number written in base ten, say 236, to a number written in another base, say base five. We know that the places in base five are the following:

$$\begin{array}{cccc} \dots & \dots & \dots & \dots \\ \text{one-hundred-} & \text{twenty-fives } (5^2) & \text{fives } (5^1) & \text{ones } (5^0) \\ \text{twenty fives } (5^3) & & & \end{array}$$

Solution: (You may find these steps easier to follow by dropping the ten subscript for now, for numbers in base ten.)

1. Look for the highest power of 5 in the base ten number; here it is 5^3 because 5^4 is 625_{ten} and 625_{ten} is larger than 236_{ten} . Are there any 5^3 s in 236_{ten} ? Yes, just one 5^3 because $5^3 = 125$, and there is only one 125 in 236. Place a 1 in the first place above to indicate one 5^3 . Now you have "used up" 125, so subtract: $236_{\text{ten}} - 125_{\text{ten}} = 111_{\text{ten}}$.
2. The next place value of five is 5^2 . Are there any twenty-fives in 111_{ten} ? There are 4, so place a 4 above 5^2 . Now four twenty-fives, or 100, has been "used", and $111_{\text{ten}} - 100_{\text{ten}} = 11_{\text{ten}}$.
3. The next place value is 5^1 which is 5. How many fives are in 11_{ten} ? It has two fives, so place 2 above 5^1 . There is 1 one left, so place a 1 above 5^0 . Thus $236_{\text{ten}} = 1421_{\text{five}}$.

MODELS FOR ADDITION & SUBTRACTION

Addition Models

ADDITIVE COMBINATION / PART + PART = WHOLE

Only 4 cars and 2 trucks were in the lot. How many vehicles were there altogether?

*JOIN MODEL:

Four girls were in a car. Two more got in. How many girls were there then?

MISSING ADDEND

Josie needs to make 15 tacos for lunch. She has made 7 already. How many more tacos does she need to make

Subtraction Models

TAKE-AWAY

Josie made 15 tacos for her friends at lunch. They ate 7 of them. How many tacos did they still have?

WHOLE – PART = PART

Of Josie's 15 tacos, 7 had chicken, and the rest had beef. How many of the tacos had beef?

COMPARISON SUBTRACTION

Josie made 15 tacos and 7 enchiladas. How many more tacos than enchiladas did Josie make?

MODELS FOR MULTIPLICATION & DIVISION

Multiplication Models

REPEATED ADDITION

Karla invited 4 friends to her birthday party. Instead of receiving gifts, she gave each of her friends a dozen roses. How many roses did she give away?

$$4 \times 12 = 12 + 12 + 12 + 12$$

(Note the order here: 4×12 means 4 groups of 12)

ARRAY / AREA MODEL

The classroom contained 6 rows of desks with 5 desks in each row. How many desks were there in the room?

A rectangle measures $3 \frac{1}{2}$ inches across and $2 \frac{1}{4}$ inches long. What is the total number of square inches contained in the rectangle?

FRACTIONAL PART OF A WHOLE

Your mother put 6 cookies on a plate. She said that you could eat $\frac{1}{2}$ of them now. How many cookies can you eat?

(this operation is $\frac{1}{2} \times 6$; how would that be different from $6 \times \frac{1}{2}$?)

CARTESIAN PRODUCT / FUNDAMENTAL COUNTING PRINCIPLE

You have 3 shirts and 2 pairs of pants in your suitcase, all color compatible. How many different outfits can you make?

An ice cream shop offers 12 choices of flavors of ice cream and 8 kinds of toppings. How many choices of double-decker ice cream cones are possible? Does your answer include two dips of the same flavor?

Division Models

REPEATED SUBTRACTION

Danielle has 12 cookies. She can fit 4 cookies in a bag. How many bags of cookies can she make?

PARTITIVE / SHARING

Danielle has 12 cookies. She wants to pass them out equally to herself and her 3 friends. How many cookies does each girl get?

(Note the difference between these two methods, how is the grouping done?)

MISSING FACTOR

Both repeated subtraction and sharing problems can be solved using this method. You just think about a missing factor, that when multiplied by the divisor, will result in the dividend.

A strange coincidence. Six hens each weigh the same. Their total weight is 42 pounds. How much does each hen weigh?
(This could also be solved by sharing)

Dividing by zero

Most of us know the rule that you cannot divide by zero. How do you know this is true?

Think about the repeated subtraction or sharing problems above using zero. Do they work? Can you make an argument as to why dividing by zero does not make sense?

Think about these division calculations and the related multiplication facts.

1. $5 \div 0 = n$. What number, if any, checks?
2. $0 \div 0 = n$. What number, if any, checks?
3. A child says, "Wait, I know that any number divided by itself is 1. So isn't $0 \div 0 = 1$?"

In base ten we could add $351 + 250$ in these two ways. (There are other ways, of course.) The first way is called the standard algorithm, and is probably the one you were taught. The second way, called an *expanded* algorithm is now being taught in some schools. Note how place value is attended to here, whereas in the standard algorithm, each "column" is treated the same: $1 + 0$, $5 + 5$, and $3 + 2 + 1$. The expanded algorithm instead considers $1 + 0$, $50 + 50$, and $300 + 200$. After the expanded algorithm is understood, it can be condensed into the standard algorithm.

$$\begin{array}{r} 1 \\ 351 \\ + 250 \\ \hline 601 \end{array}$$

$$\begin{array}{r} 351 \\ + 250 \\ \hline 1 \text{ (thinking } 1 + 0) \\ 100 \text{ (thinking } 50 + 50) \\ \underline{500} \text{ (thinking } 300 + 200) \\ 601 \end{array}$$

We can also use either method for adding in other bases, but the second method is sometimes easier to follow until adding in another base is well understood.

Example: Here is an example using both the standard and expanded algorithms to add the same two numbers in base eight. Make sure you can understand each way.

$$\begin{array}{r} 1 \\ 351_{\text{eight}} \\ + 250_{\text{eight}} \\ \hline 621_{\text{eight}} \end{array}$$

$$\begin{array}{r} 351_{\text{eight}} \\ + 250_{\text{eight}} \\ \hline 1_{\text{eight}} \text{ thinking } (1 + 0) \\ 120_{\text{eight}} \text{ thinking } (50 + 50) \\ \underline{500_{\text{eight}}} \text{ thinking } (300 + 200) \\ 621_{\text{eight}} \end{array}$$

Activity: Adding in Base Four

Add these two numbers in base four in both expanded and standard algorithms: 311_{four} and 231_{four} .

If we can add in different bases, we should be able to subtract in different bases. Here is an example of how to do this.

Example: Find $321_{\text{five}} - 132_{\text{five}}$. One way to think about this problem is to regroup in base five just as we do in base ten, then use the standard way of subtracting in base ten.

Solution:

$$\begin{array}{r} 321 \\ - 132 \\ \hline \end{array}$$

Step 1: We cannot remove 2 ones from 1 one, so we need to take one of the fives from 321_{five} and change it to five ones:

$$321_{\text{five}} \rightarrow 300_{\text{five}} + 20_{\text{five}} + 1_{\text{five}} \rightarrow 300_{\text{five}} + 10_{\text{five}} + 11_{\text{five}}$$

Step 2: We can now take 2 ones from 11 ones (in base five) leaving 4 ones. (Notice how 321 has changed with 3 five squared, then 1 five, then 11 ones, from Step 1.)

$$\begin{array}{r} 1 \\ 3 \quad \underline{2} \quad 11_{\text{five}} \\ - 1 \quad 3 \quad 2_{\text{five}} \\ \hline 1 \quad 3 \quad 4_{\text{five}} \end{array}$$

means 3 (five squared) + 1 five + 11 ones as in Step 1.

Step 3. In the fives place: We cannot subtract 3 fives from 1 five, so we must change one five squared to five sets of five. This, together with the one five already in place, we know have 11 fives.

That is: $300_{\text{five}} + 10_{\text{five}} \rightarrow 200_{\text{five}} + 110_{\text{five}}$, so

$$\begin{array}{r} 2 \quad 11 \\ 3 \quad \underline{2} \quad 11_{\text{five}} \\ - 1 \quad 3 \quad 2_{\text{five}} \\ \hline 1 \quad 3 \quad 4_{\text{five}} \end{array}$$

means 11 fives, not 11 ones, so the 11 stands for 110 and 11 fives minus 3 fives is 3 fives, or $110 - 30$ is 30

We now have 2 (five squared) from which 1 (five squared) is subtracted, leaving 1 (five squared). The answer is 1 five-squared plus 3 fives plus 4 ones which is 134_{five} .

Activity: Subtracting in Base Four

Subtract 231_{four} from 311_{four} in base four.