

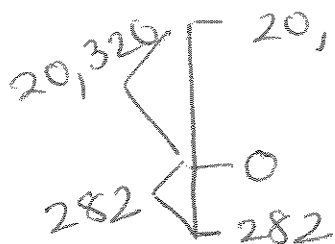
Learning Exercises for Section 1.2

- Some problems are simple enough that the quantitative structure is obvious, particularly after a drawing is made. The following problems are from a fifth-grade textbook.ⁱ For each problem given, make a drawing and then provide the answer to the problem.
 - The highest elevation in North America is Mt. McKinley, Alaska, which is 20,320 feet above sea level. The lowest elevation in North America is Death Valley, California, which is 282 feet below sea level. What is the change in elevation from the top of Mt. McKinley to Death Valley?

Quantities

Height: Mt McKinley
 Height: Death Valley
 Difference in heights

20,320 above
 282 below
 ??

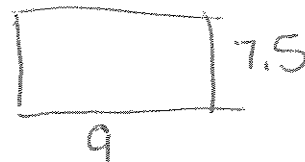


$$\begin{array}{r}
 20,320 \\
 + \quad 282 \\
 \hline
 20,602 \text{ difference}
 \end{array}$$

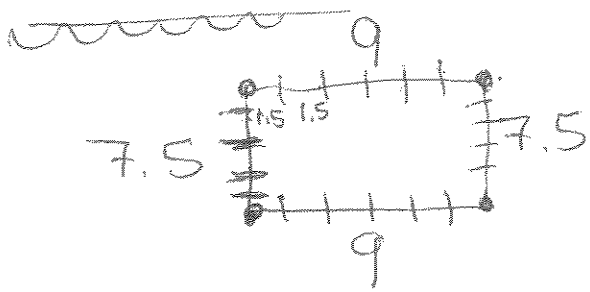
- These problems are from a sixth-grade textbookⁱⁱ from a different series. This time, undertake a full quantitative analysis to solve each of the problems.
 - Kelly wants to fence in a rectangular space in her yard, 9 meters by 7.5 meters. The salesperson at the supply store recommends that she put up posts every 1.5 meters. The posts cost \$2.19 each. Kelly will also need to buy wire mesh to string between the posts. The wire mesh is sold by the meter from large rolls and costs \$5.98 a meter. A gate to fit in one of the spaces between the posts costs \$25.89. Seven staples are needed to attach the wire mesh to each post. Staples come in boxes of 50, and each box costs \$3.99. How much will the materials cost before sales tax?

Quantities

length of yard = 9 m
 width of yard = 7.5 m
 perimeter of yard = ?
 distance between posts = 1.5 m
 number of posts needed = ?
 cost of a post = 2.19
 Total cost of posts = ?
 cost of a meter of fence = 5.98
 Total cost of fence = ?
 cost of gate = 25.89
 cost of staples (box) = 3.99



staples per fence post = 7
 total number of boxes of staples = ?
 total cost of staples = ?
 total cost of materials = ?



perimeter of yard
 $= 18 + 15 = 33 \text{ m}$

need a post at each corner and every 1.5 m between
 so need 22 posts (see picture)

(OR $33 \div 1.5 = 22$ posts)

cost of posts = $(22)(2.19) = \boxed{\$48.18}$
 gate costs $\boxed{\$25.89}$ and takes $1\frac{1}{2} \text{ m}$ of fence
 need $3\frac{1}{2} \text{ m}$ of fence but sold by meter
 cost of fence = $(32)(5.98) = \boxed{\$191.36}$

need 7 staples per post for
 22 posts $(22)(7) = 154$ staples
 sold in boxes of 50 so need 4 boxes
 cost of staples = $(4)(3.99) = \boxed{\$15.96}$

Total cost = $48.18 + 25.89 + 191.36 + 15.96$
 $= \$281.39$

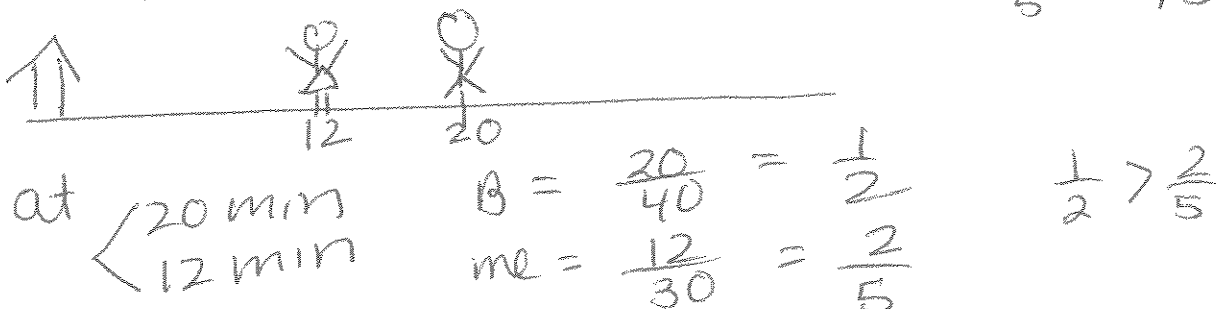
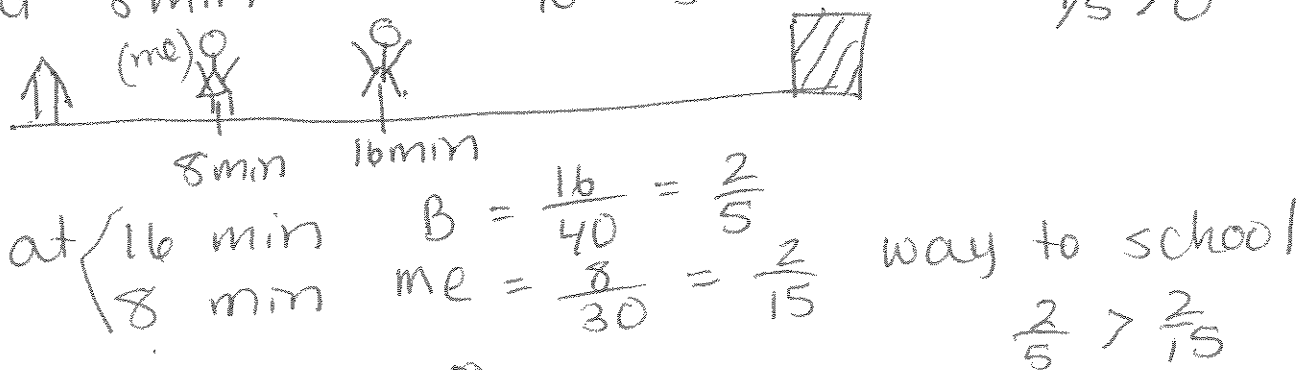
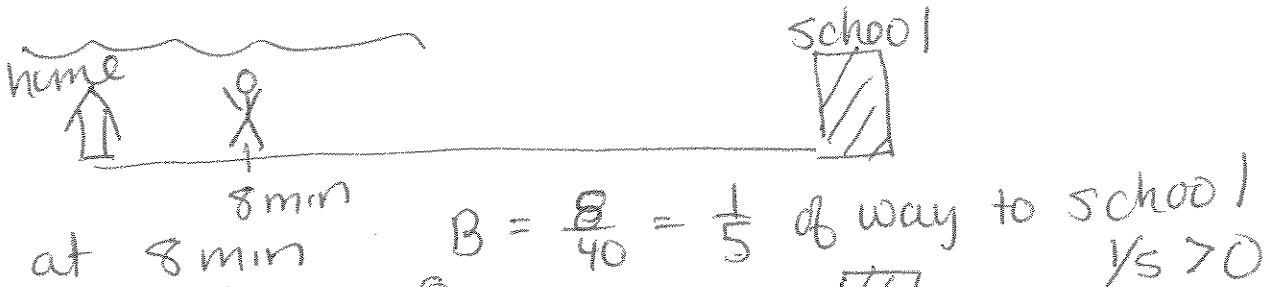
4. My brother and I walk the same route to school every day. My brother takes 40 minutes to get to school and I take 30 minutes. Today, my brother left 8 minutes before I did.

- How long will it take me to catch up with him?
- Part of someone's work on this problem included $\frac{1}{30} - \frac{1}{40}$. What quantities do the two fractions in $\frac{1}{30} - \frac{1}{40}$ represent?
- Suppose my brother's head start is 5 minutes instead of 8 minutes. Now how long does it take for me to catch up with him?

Quantities

speed brother
 speed me
 difference in starting times
 brother time to school
 my time to school
 distance to school
 difference in speeds
 amount of time to catch up
 distance between brother & me
 (at some time)

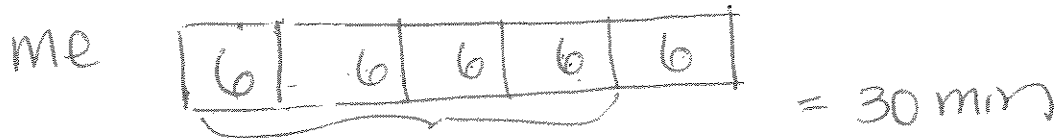
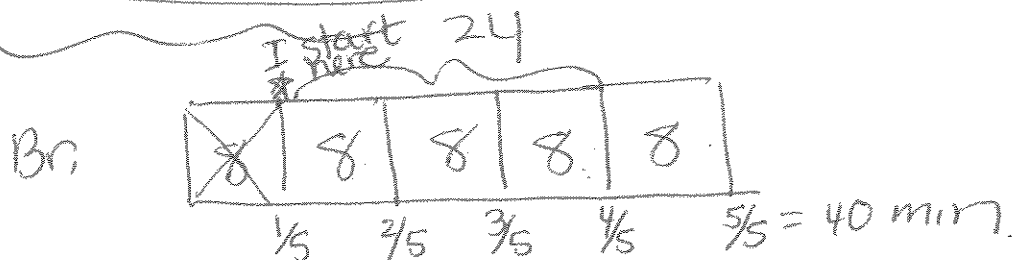
8 min
 40 min
 30 min



at 24 min 16 min $B = \frac{24}{40} = \frac{3}{5}$ $me = \frac{16}{30} = \frac{8}{15}$ $\frac{3}{5} > \frac{8}{15}$

at 28 min 20 min $B = \frac{28}{40} = \frac{7}{10}$ $me = \frac{20}{30} = \frac{2}{3}$ $\frac{7}{10} > \frac{2}{3}$

at 32 min 24 min $B = \frac{32}{40} = \frac{4}{5}$ $me = \frac{24}{30} = \frac{4}{5}$ same



24 min from when I start (or 32 from brothers start we will be in same place)

Brother is 8 min ahead of me.
(which is $\frac{1}{5}$ of the trip)
Each min I walk $\frac{1}{30}$ of the distance
and my brother walks $\frac{1}{40}$ of the distance
to school, so I catch up by

$$\frac{1}{30} - \frac{1}{40} = \frac{4}{120} - \frac{3}{120} = \frac{1}{120}$$

$$\frac{1}{120} (x) = \frac{1}{5}$$

$x = 24 \text{ min}$ for me to catch up

6. The last part of one triathlon is a 10K (10 kilometers, or 10,000 meters) run. When runner Aña starts this last running part, she is 600 meters behind runner Bea. But Aña can run faster than Bea: Aña can run (on average) 225 meters each minute, and Bea can run (on average) 200 meters each minute. Who wins, Aña or Bea? If Aña wins, when does she catch up with Bea? If Bea wins, how far behind is Aña when Bea finishes?

Quantities

Length of race (part) = 10,000 m

How far Aña has to run

How far Bea has to run

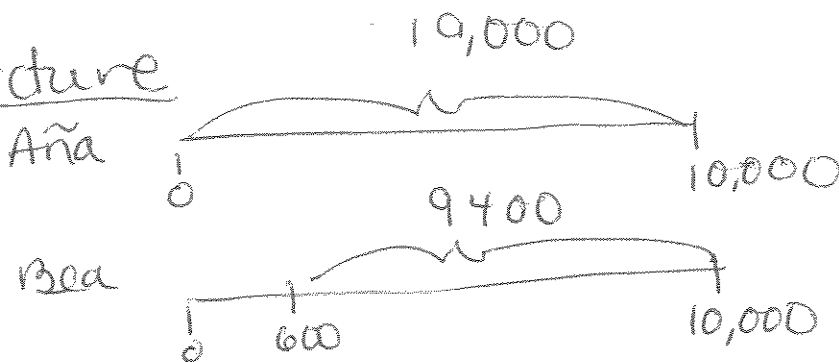
Aña speed

Bea speed

Difference in speeds

Difference in distances when Aña starts running

Picture



$$\text{Aña} \quad 10,000 \div 225 = 44.4 \text{ minutes}$$

$$\text{Bea} \quad 9400 \div 200 = 47 \text{ minutes}$$

So Aña will beat Bea

$$\text{Difference in distance} = 600$$

$$\text{Difference in speed} = 25 \text{ m/min}$$

$$600 \div 25 = 24 \text{ minutes they will tie}$$

$$(225 \times 24 = 5400 \text{ m})$$

$$(200 \times 24 + 600 = 4800 + 600 = 5400 \text{ m})$$

after Aña reaches 5400 m

7. Research on how students solve word problems contained the following incident.^{iv} Dana, a seventh grader in a gifted program in mathematics, was asked to solve the following problem:

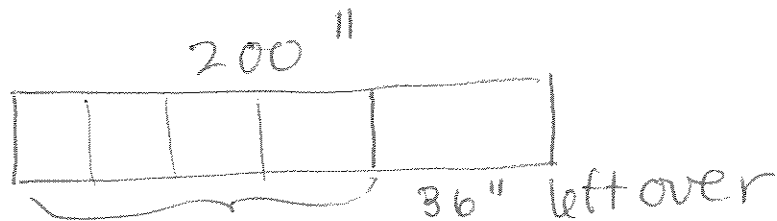
◆ A carpenter has a board 200 inches long and 12 inches wide. He makes 4 identical shelves and still has a piece of board 36 inches long left over. How long is each shelf? ◆

Dana tried to solve the problem as follows: She added 36 and 4, then scratched it out, and wrote 200×12 , but she thought that was too large so she scratched that out. Then she tried $2400 - 36$ which was also too large and discarded it. Then she calculated 4×36 and subtracted that from 200, getting 56. She then subtracted 12, and got 44.

Dana used a weak strategy called "Try all operations and choose." She obviously did not know what to do with this problem, although she was very good at solving one-step problems.

Do a quantitative analysis of this problem situation, and use it to make sense of the problem in a way that Dana did not. Use your analysis to solve the problem.

<u>Quantities</u>		
* length of board		200 in
(not used) * width of board		12 in
* left over board		36 in
* 4 identical pieces		? in.
* board to be cut in 4		? in.



cut in 4 = ?
pieces = ?

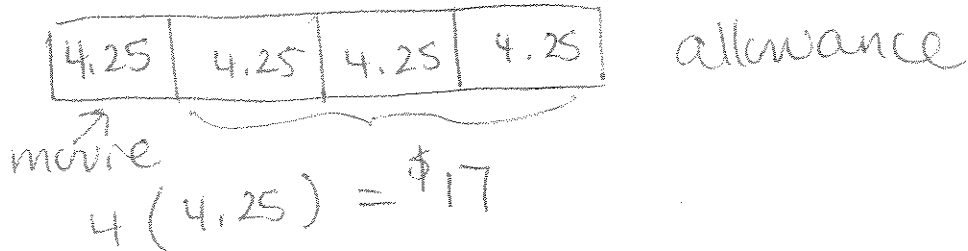
$$200 - 36 = 164 \text{ " to be cut into shelves}$$

$$164 \div 4 = 41 \text{ " for each shelf}$$

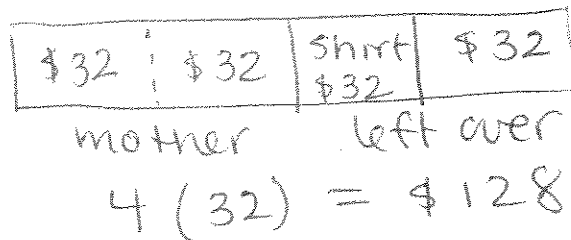
Learning Exercises for Section 1.4

Use a mature strategy and a strip diagram to work each of the following:

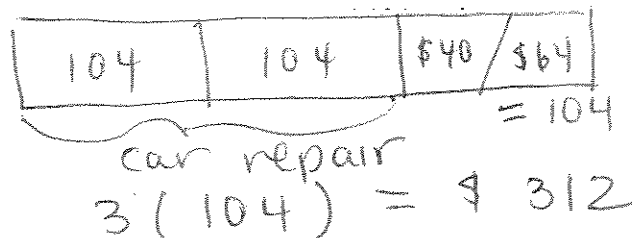
1. Kalia spent a quarter of her weekly allowance on a movie. The movie was \$4.25. What is her weekly allowance?



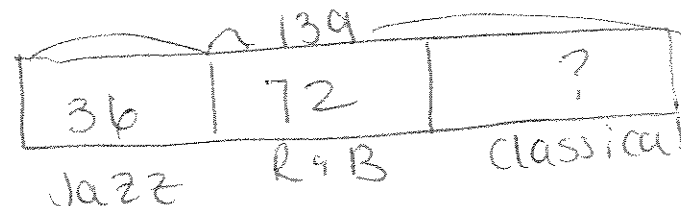
3. Nghiep gave his mother half of his weekly earnings, and then spent half of what was left on a new shirt. He then had \$32. What were his weekly earnings?



6. Jinfa, upon receiving his paycheck, spent two-thirds of it on car repairs and then bought a \$40 gift for his mother. He had \$64 left. How much was his paycheck?



9. Jo-Jo has downloaded 139 songs on his iPod. Of those songs, 36 are jazz, twice that are R & B, and the remaining are classical. How many classical songs has he downloaded?



$$36 + 72 = 108$$

$$139 - 108 = 31 \text{ classical}$$