Learning Exercises for Section 15.1

1. Give the next four entries, and the other ones requested, as suggested by these patterns if they continue indefinitely: odd = A , even = B

   a. ABABAB A B A B and the 100th entry. B

   b. ABBAABBA A B B A and the 63rd entry. B mult. of 4 ABBA

   c. 6.5, 7.3, 8.1, 8.9, 9.7, 10.5, 11.3, 12.1 and the 20th entry. 21.7

   d. 100, 95, 90, 85, 80, 75, 70, 65 and the 30th entry. -45

   e. 2, 6, 18, 54, 162, 486, 1458, 4374

   f. 5, 2.5, 1.25, 0.625, .3125, .15625, .078125, .0390625

   g. $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$ and the 100th entry. $\frac{1}{100}$

   h. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$ and the 100th entry. $\frac{100}{101}$

   i. 2, 4, 6, 8, 2, 4, 6, 8, 2, 4, 6, 8 and the 30th entry. 4

   j. 1, 4, 2, 8, 5, 7, 1, 4, 2, 8, 5, 7, 1, 4, 2, 8, 5, 7 and the 40th entry. 8

   k. Which of the sequences in a-j are arithmetic sequences? C & D

   l. Which, if any, are geometric sequences? E & F

2. a. Examine these (correct) calculations and look for an easy rule for multiplying by a power of 10. Write the rule down.

12.3457 x 10 = 123.457	23 x 10 = 230
12.3457 x 100 = 1234.57	23 x 100 = 2300
12.3457 x 1000 = 12345.7	23 x 1000 = 23 000
12.3457 x 10 000 = 123457.	23 x 10 000 = 230 000
12.3457 x 100 000 = 1234570.	23 x 100 000 = 2 300 000

   Count the zeros in the power of 10 (or look at the exponent if scientific notation 10^3) Move the decimal place that many places to create the larger number

   b. Why does the rule work?

   It works because our decimal place system works off powers of 10. So each digit moves that many places to the left.

3. a. Examine these (correct) calculations and look for an easy rule for dividing by a power of 10. Write the rule down.

$512.345 \div 10 = 51.2345$	$8.41 \div 10 = 0.841$
$512.345 \div 100 = 5.12345$	$8.41 \div 100 = 0.0841$
$512.345 \div 1000 = 0.512345$	$8.41 \div 1000 = 0.00841$
$512.345 \div 10\,000 = 0.00512345$	$8.41 \div 10\,000 = 0.000841$
$512.345 \div 100\,000 = 0.000512345$	

count how many zeros there are in the power of 10. Move the decimal place that many places to make a smaller number

b. Why does the rule work?

Each place value is reduced by that power so each digit moves to the right that many places

c. How can this idea be used to find 1% of a known amount? 10%?

$1\% = \frac{1}{100}$ or divide by 100

$10\% = \frac{10}{100} = \frac{1}{10}$ or divide by 10

4. Find likely function rules for the following, and find the missing values for n.

a.

x	y
1	9
2	30
3	51
4	72
...	...
n	1038

b.

x	f(x)
1	78
2	56
3	34
4	12
...	...
n	$^-120$

c

input	output
1	0
2	8
3	16
4	24
...	...
n	1608

$y = 21x - 12$

$n = 50$

$f(x) = 100 - 22x$

$n = 10$

output = 8 x inp −8

$n = 76$

d.

input	output
50	8
51	8.3
52	8.6
53	8.9
...	...
n	15.8

e.

x	g(x)
1	1
2	4
3	9
4	16
...	...
n	900

f.

x	h(x)
1	2
2	6
3	12
4	20
5	30
...	...

d. output $= 0.3 \times$ input $- 8$

$n = 76$

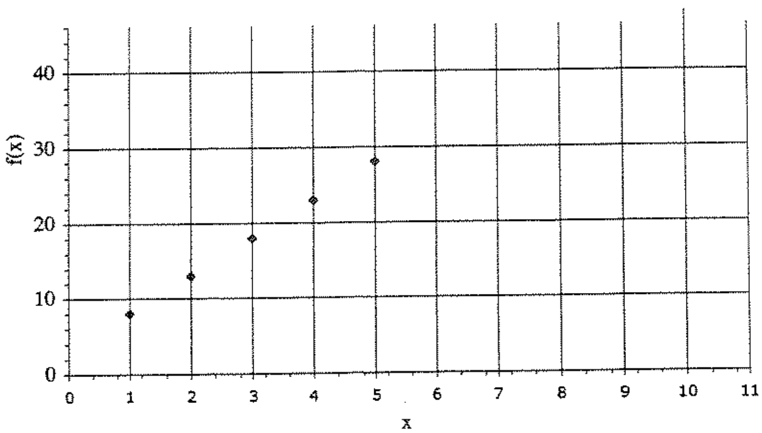
f. $h(x) = x(x+1)$

$= x^2 + x$

e. $g(x) = x^2$

$n = 30$

g.



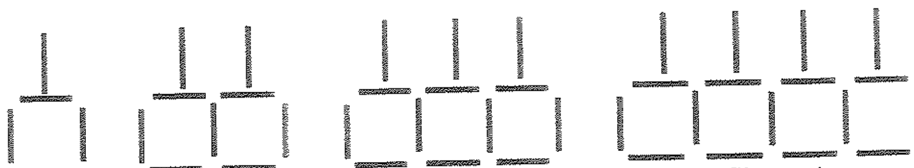
$f(x) = 5x + 3$

h. Which of a-g are examples of arithmetic sequences?

a, b, c, d, g

5. Find a function rule for each of the following, and justify that your rule will be true in general.

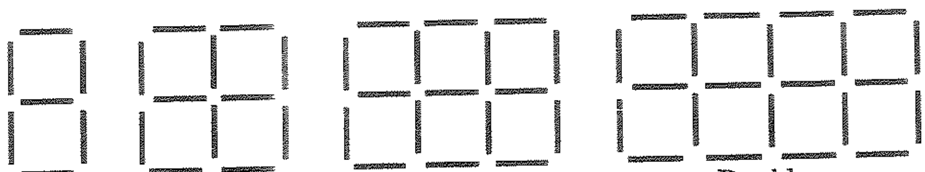
a. The number of toothpicks to make Shape n in the pattern:



Shape 1 Shape 2 Shape 3 Shape 4

$f(n) = 4n+1$

b. The number of toothpicks to make Double-decker n:



Double-decker 1 Double-decker 2 Double-decker 3 Double-decker 4

$f(n) = 5n+2$

c. The number of toothpicks to make Shape n:

Shape 1 Shape 2 Shape 3 Shape 4

$f(n) = 2n+1$

d. The number of toothpicks to make Shape n:

Shape 1 Shape 2 Shape 3

$f(n) = 3n+4$

e. The number of toothpicks to make row-house n:

Row-house 1 Row-house 2 Row-house 3

$f(n) = 5n+6$

f. Make up a toothpick pattern of your own, and challenge others to find a rule for it.

8. The numbers 1, 1, 2, 3, 5, 8, ... are an example of a Fibonacci (fi-ba-NAH-chee) sequence, a pattern that appears in nature, art, and geometry.

a. What are the next four numbers in that Fibonacci sequence? (Hint: Look at two, then the next one.)

1, 1, 2, 3, 5, 8, 13, 21, 34, 55

b. Amazingly, the nth number in that Fibonacci sequence is $\frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$! Verify that for n = 1 and n = 2, that expression does give the first two numbers in the pattern, 1 and 1.

$n=1$ $\quad \frac{(1+\sqrt{5})^1 - (1-\sqrt{5})^1}{2^1\sqrt{5}} = \frac{1+\sqrt{5}-1+\sqrt{5}}{2\sqrt{5}} = \frac{2\sqrt{5}}{2\sqrt{5}} = 1$

$n=2$ $\quad \frac{(1+\sqrt{5})^2 - (1-\sqrt{5})^2}{2^2\sqrt{5}} = \frac{1+2\sqrt{5}+5-1+2\sqrt{5}-5}{4\sqrt{5}}$

$= \frac{4\sqrt{5}}{4\sqrt{5}} = 1$

c. (Calculator) Give decimals for the first ten ratios of consecutive Fibonacci numbers: $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$ The value the ratios get closer and closer to is called the golden ratio.

the ratios approach 1.61

(actual value $\frac{1+\sqrt{5}}{2}$)

14. a. What digit is in the ones' place in the calculated form of 3^{250}?
b. What digit is in the ones' place in the calculated form of 7^{350}?

a)

# of events	
1	$3^1 = 3$
2	$3^2 = 9$
3	$3^3 = 27$
4	$3^4 = 81$
5	$3^5 = 243$
6	$3^6 = 729$
7	$3^7 = 2187$
8	$3^8 = 6561$

pattern 3, 9, 7, 1, 3, 9, 7, 1 . . .

250^{th} term = ends 9

b)

# of events	
1	$7^1 = 7$
2	$7^2 = 49$
3	$7^3 = 343$
4	$7^4 = 2401$
5	$7^5 = 16807$
6	$7^6 = 117649$
7	$7^7 = 823543$
8	$7^8 = 5764801$

pattern 7, 9, 3, 1, 7, 9, 3, 1:

350^{th} term ends in 9

c. What digit is in the ones' place in the calculated form of $3^{250} \cdot 7^{350}$?

3^{250} ends in 9
7^{350} ends in 9
so product ends in $9 \times 9 = 8\underline{1}$

d. Write down how you would proceed, to find the digit in the ones' place in the calculated form of 7^n.

pattern is 7, 9, 3, 1 so find how many full multiples of 4 fit into n, then count out any remaing digits in remainder for placement of ones place digit

22. What digit is in the 99th decimal place in the decimal for $\frac{1}{7}$? Explain your reasoning.

1 2 3 4 5 6

$$
\begin{array}{r}
.142857 \\
7\overline{)1.00000} \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
1
\end{array}
$$

6 repeating digits

$$
\begin{array}{r}
16 \\
6\overline{)99} \\
\underline{6} \\
39 \\
\underline{36} \\
3
\end{array}
$$

3 left over

so 99th decimal place is a 2

25. Figure out a shortcut for squaring a number ending in 5.

$5^2 = 25$

$15^2 = 225$ $2 = 1 \times 2$ always ends in 25

$25^2 = 625$ $6 = 2 \times 3$

$35^2 = 1225$ $12 = 3 \times 4$

$45^2 = 2025$ $20 = 4 \times 5$

beginning digits are the number formed by digits before last 5 time a number one larger

ie

$1005^2 = (100 \times 101)$ with 25 on end

$= 1010025$

Learning Exercises for Section 15.2

1. Explain how these every-day sentences make sense, mathematically (if the characteristics are quantifiable).

   a. "Sales of a CD is a function of the singer's popularity."

If popularity could be assigned a number, then as popularity increased numerically, sales would increase

   b. "Success is a function of how much you work."

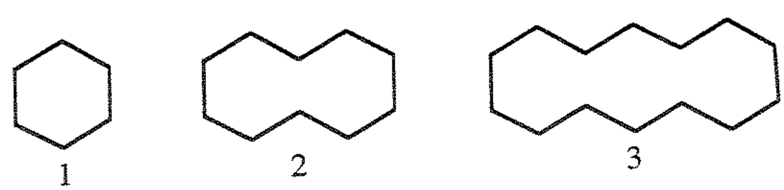
as the number of hours worked increased so would success (if you could assign it a number)

   c. "Reaction time is a function of blood alcohol content."

As blood alcohol levels change, so does reaction time - and knowing a blood alcohol level, you can predict reaction time.

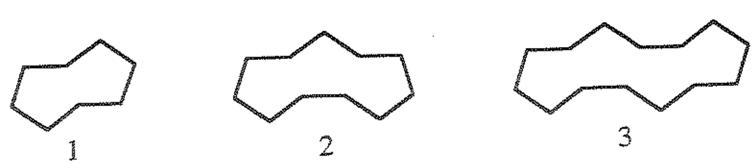
2. Find a function rule that gives the perimeter of the nth shape in each of the following patterns. Justify that your rule will always work. (Use the shortest segment as the measuring unit.)

a.



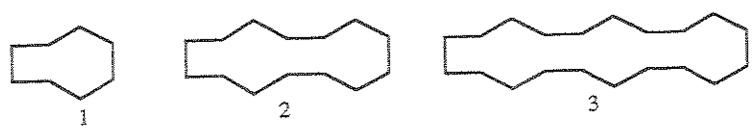
$P(n) = 4n+$

b.



$P(n) = 3n+5$

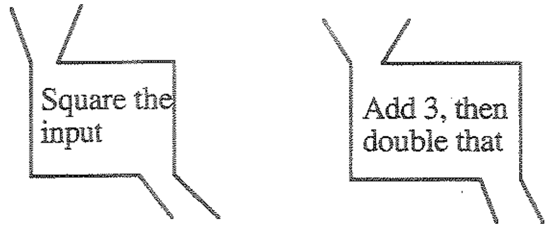
c.



$P(n) = 6n+$

3. For each input give the output for the combination, first Machine 1, then Machine 2.

Machine 1 Machine 2



a. Input = 5 b. Input = 10 c. Input = x

56

206

$2(x^2+3)$
$= 2x^2+6$

d-f. Repeat with the part a inputs, but for the combination, first Machine 2, then Machine 1.

d. Input = 5
256

e. Input = 10
676

f. Input = x
$[2(x+3)]^2$
$= 4(x+3)^2$