Exercises for Section 12.1 (1-3)

* 1. The following situations were written by public elementary school students. For each situation, write a sentence that shows how two quantities in the situation are related. There may be several relationships that you can describe in each situation.

Example: *A large block of ice is on the sidewalk in the sun. It melts 3 centiliters a minute.*

The amount of ice decreases as the number of minutes that pass increases.
As time passes, the amount of water on the sidewalk increases.

a. An airplane is flying miles high over Siberia, hundreds of miles from any help. Suddenly, the tank ruptures and gas starts steadily leaking out.

b. A motor home has a water tank that holds a large amount of water. Each shower takes 5 gallons of water.

c. You have \$5 and you buy candy bars at 50¢ apiece.

d. A classroom starts off with 16 students. Two students stop coming every week.

* 2. a. Write your own story like those the students wrote in Exercise 1.

b. Then write a sentence that shows how two quantities in the situation are related. There may be several quantitative relationships that you can describe.

3. Determine whether the pairs of quantities below are related to one another. If so, explain whether the value of the quantity on the right increases or decreases as the value of the quantity on the left increases.

Problem-solving tip: Try some examples and see what happens.

a. the perimeter of a square and the area of a square

b. the base of a triangle and the area of a triangle

c. the value of the numerator in a fraction (when the denominator stays the same) and the value of the fraction

d. the value of the denominator in a fraction (when the numerator stays the same) and the value of the fraction

Exercises for Section 12.2 (1-3, 5)

* 1. **Help! Something's Leaking, Part I.** An airplane is flying high over Siberia, hundreds of miles from any help. Suddenly, the gas tank starts leaking. The tank contained 80 gallons of gas when the tank started leaking and is leaking at a rate of 10 gallons each minute. [Note: This scenario was written by a public school student.]

a. How does the amount of gas in the tank change as time passes?

decrease by 10 gallons every minute

b. Complete the following table of data describing the relationship between the amount of gas in the tank and the time spent flying since the rupture.

Time (in minutes)	Amount of gas in tank (in gallons)
0	80
1	70
3	50
4	40
6	20
8	0

c. How can you find the amount of gas in the tank if you know the number of minutes that have passed?

remaining gallons = 80 gallons − 10 (minutes)

d. If T represents the number of minutes that have passed and G represents the amount of gas in the tank, write an equation between the two quantities.

$G = 80 - 10T$

e. How can you check your equation?

f. When will the gas tank be empty? Explain how you know.

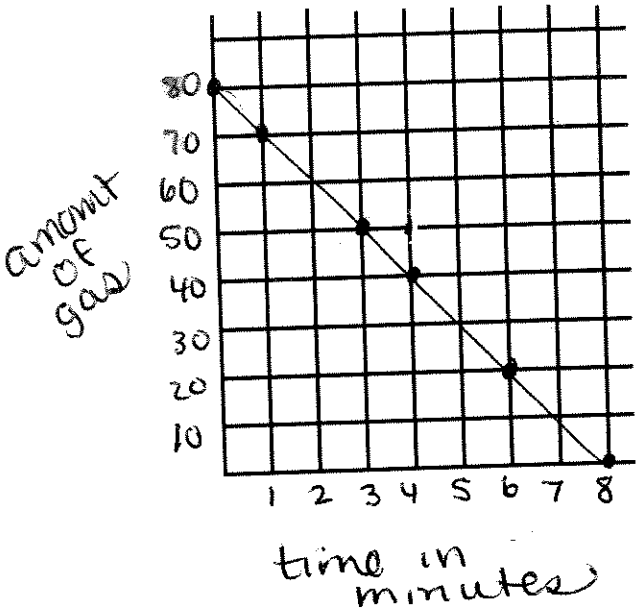
$G = 80 - 10t$ at 10 minutes

$G = 80 - 10(8)$

$G = 80 - 80$

$G = 0$

g. Graph the relationship between the amount of gas in the tank and the time spent flying since the gas leak started.



* 2. **Help! Something's Leaking, Part II.** Consider the scenario in Exercise 1. However, this time think about the relationship between the amount of gas that has leaked and the time that has passed since the rupture in the tank.

a. Describe in words how the amount of gas that has leaked changes over time.

Amount leaked increases by 10 gallons every minute

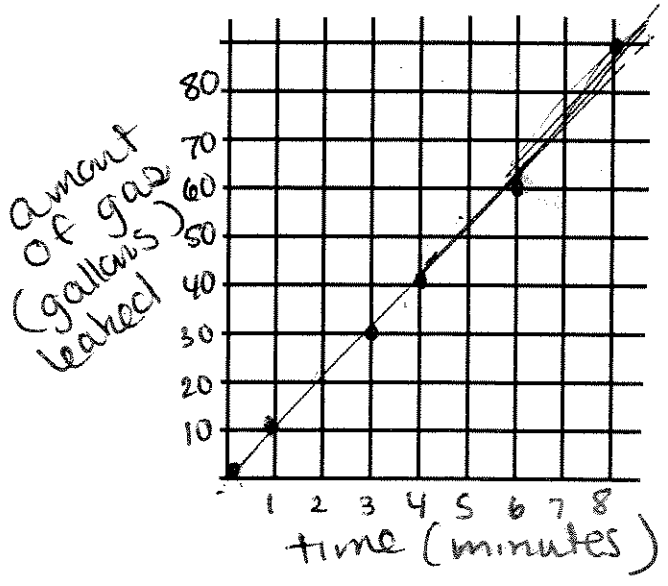
b. Create a table of data to describe the relationship between the amount of gas that has leaked and time.

time (minutes)	Amount of gas leaked (in gallons)
0	0
1	10
3	30
4	40
6	60
8	80

c. Use algebraic symbols to describe the relationship between the amount of gas that has leaked and time. Be sure to label your variables. G = gas leaked, T = time

$$G = 10T$$

d. Graph the relationship between the amount of gas that has leaked and the time spent flying. Be sure to label the quantities on each axis.



e. How do the graphs from Exercises 1g and 2d differ? What does this say about the quantitative relationships?

first graph shawed a decreasing relationship between time and gallons left in tank

this graph shawed an increasing relationship between time and gallons leaked

There are often two viewpoints for quantitative relationships, one increasing and one decreasing.

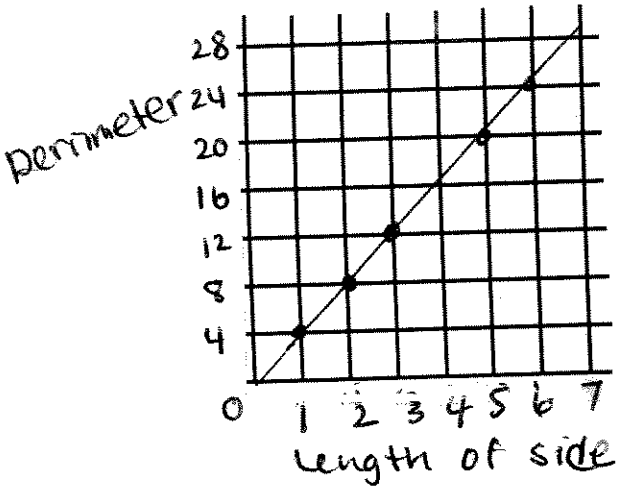
f. At what rate is the gas leaking? How can you tell using each graph (from Exercises 1g and 2d)?

It is changing by 10 gallons per minute.

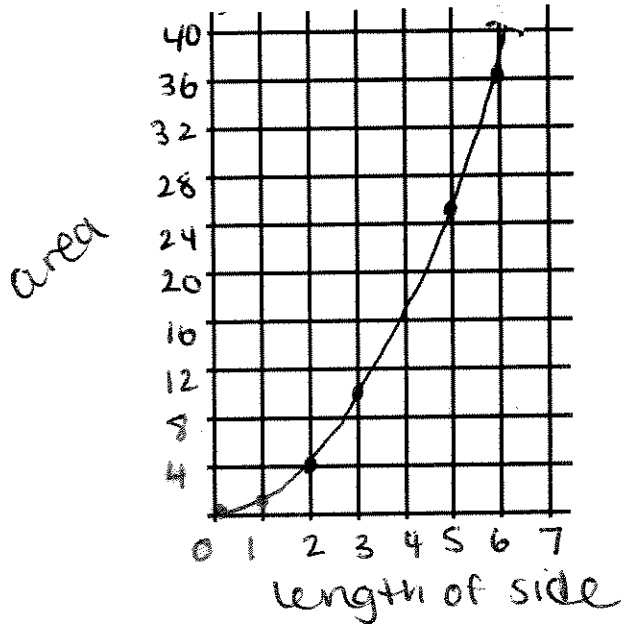
3. **Geometric Relationships.**

   a. Graph the relationship between the length of the side of a square and the perimeter of the square. You may first want to generate a table of data, using several squares.

length of side of square	Perimeter	Area
1	4	1
2	8	4
3	12	9
5	20	25
6	24	36



   b. Graph the relationship between the length of the side of a square and the area of the square.



c. Compare the two graphs. How are they alike? How are they different?

Both are increasing but perimeter is related to length by a straight line, while area is related to length by a curve

d. Let s = the length of a side of a square; p = the perimeter of a square; and A = the area of a square. Write an equation to represent the relationship between the length of the side of a square and the perimeter of the square. How can you check to make sure your equation is correct? Now write an equation to represent the relationship between the length of the side of a square and the area of the square. Compare the two equations. How are they alike? How are they different? How are the differences related to differences between the two graphs?

$P = 4S$

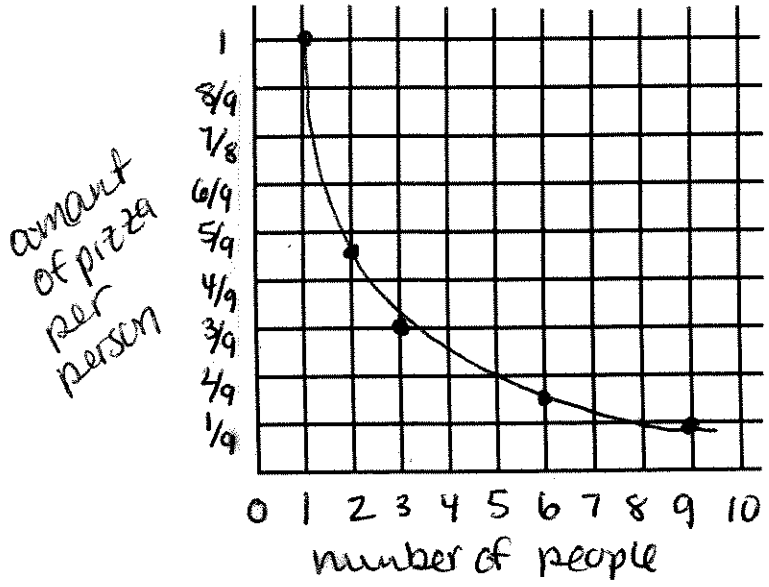
$A = S \cdot S$ or $A = S^2$

5. **Fraction Relationships.**

   a. Suppose you are sharing a pizza fairly among several people. Create a table of data that shows what fraction each person will receive for different numbers of people.

number of people	1	2	3	6	9
fraction per person	1	1/2	1/3	1/6	1/9

   b. Graph the relationship between the number of people sharing the pizza and the size of the piece each receives.



   c. Does it make sense to connect the dots? Why or why not?

we cannot have fractional amounts of people so connecting the dots may not give a totally correct picture

d. Does the size of each pizza piece increase or decrease as the number of people sharing the pizza increases?

The graph shows (and does logic) that the size of the piece of pizza decreases as the number of people increase

e. Why is the shape of this graph a curve instead of a line?

fractions are divisions

f. What, if anything, would change if the people were sharing 2 pizzas? 6 pizzas? 3/4 pizza?

the pieces would be larger, but the relationship has basically the same shape.

Exercises for Section 12.3 (1-5)

1. **Strike!**

a. Calculate the slope of the graph shown below.

Choose two points on line

2 games, $8 4 games, $14

$$\text{slope} = \frac{14-8}{4-2} = \frac{6}{2} = 3$$

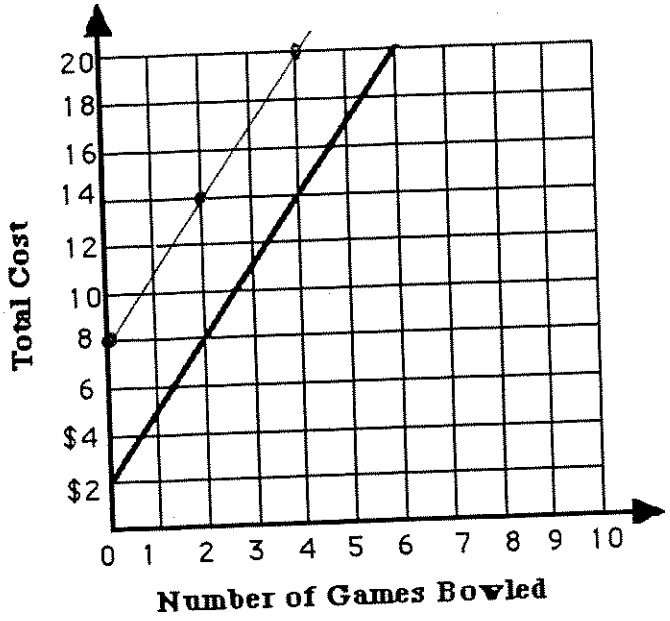
b. What does the slope mean in terms of the bowling situation?

For every $3, you can bowl one game

Bowling costs $3 per game

(* after initial $2 fee)

c. Draw another line on the graph that has the same slope. What does this line represent in terms of the bowling situation?



New line represents $8 fee and $3 per game after that

2. **Help! Something's Leaking, Part 3.** Look back at Exercises 1 and 2 from Section 2.1. Both exercises dealt with the situation of an airplane flying over Siberia with a leaky gas tank.

a. Calculate the slope of the graph that you created in Exercise 1g. This graph showed the amount of gas in the tank over time.

$$\text{Slope} = \frac{20-40}{6-4} = \frac{-20}{2} = \frac{-10}{1} = -10$$

b. What does this slope tell you about the situation with the airplane?

The amount of gas in plane decreasing by 10 gallons per minute

c. Calculate the slope of the graph that you created in Exercise 2d. This graph showed the amount of gas that had leaked over time.

$$\text{slope} = \frac{40-20}{4-2} = \frac{20}{2} = \frac{10}{1} = 10$$

d. What does this slope tell you about the situation with the airplane?

The amount of gas leaking from plane increasing by 10 gallons per minute

e. Compare the two slopes in parts a and c. How are they alike and why? How are they different and why?

Both describe 10 gallon change per minute

One is a decreasing relationship

one is an increasing relationship

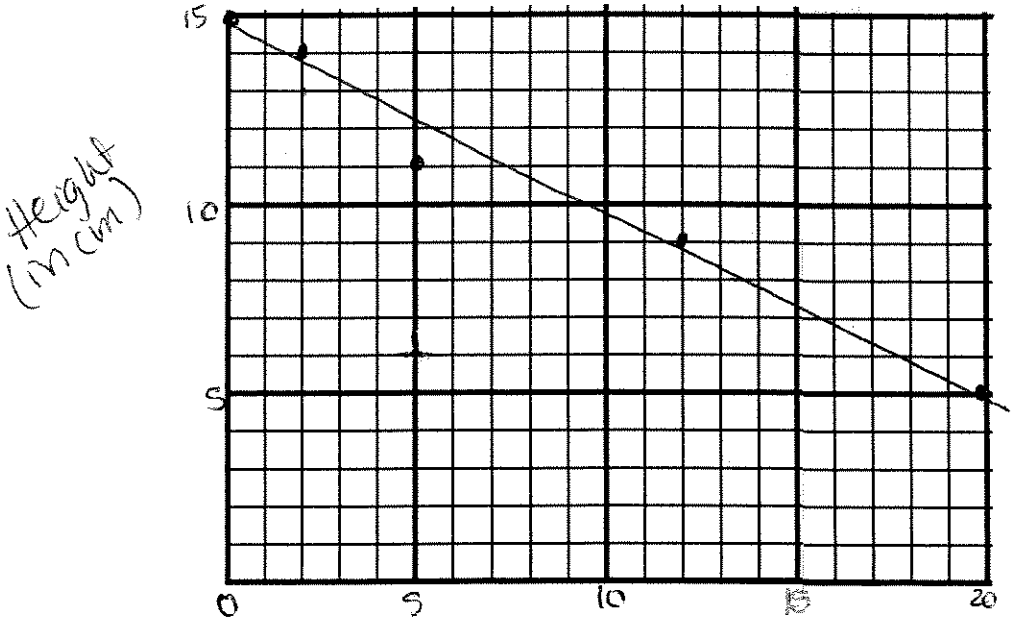
3. **Burning Candles, Part 1.** The following data were collected as a candle burned. The data show how the height of the candle changed over time.

Number of minutes a candle has burned	Height of the candle in centimeters
0	15
2	14
5	11
12	9
20	5

a. Graph the data. Put height of the candle on the vertical axis and time on the horizontal axis. smooth line

b. Calculate the slope of the line.

c. What does the slope tell you about the candle?



$$\text{slope} = \frac{15-5}{0-20} = \frac{10}{-20} = -\frac{1}{2}$$

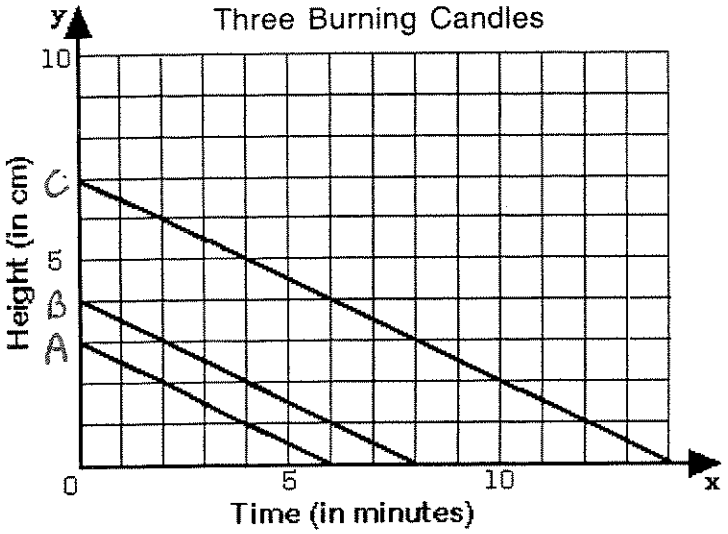
height is decreasing by about 1/2 cm per minute

height = 15 − 1/2 time

$H = -\frac{1}{2}T + 15$

$y = mx + b$ form

4. **Burning Candles, Part 2.**



a. What was the starting height of each candle?

Candle C - 7 cm
Candle B - 4 in
Candle A - 3 in

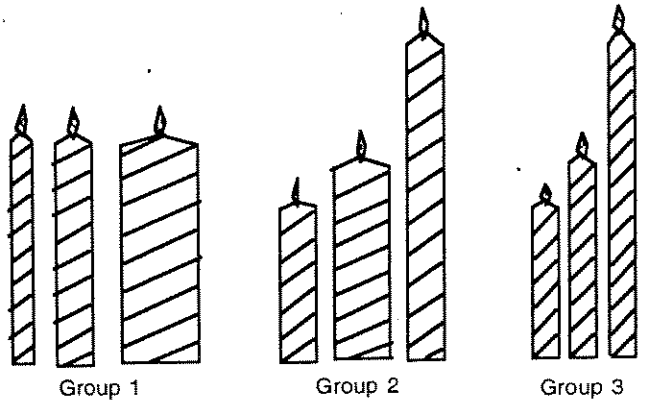
b. At what rate did each candle burn? How do you know?

All burned at same rate because slopes are the same 1 in every 2 minutes

c. What is the slope of each line?

slope = $\frac{1}{2}$

d. Which of the following groups of candles could be represented by the graph and why? Which candle goes with which line?



4 cont.

e. Write an equation to represent each candle's burning. (Make a table of data for each candle first, if it will help.)

Candle A $H = 3 - \frac{1}{2}T$

Candle B $H = 4 - \frac{1}{2}T$

Candle C $H = 7 - \frac{1}{2}T$

f. How are the three equations alike? How are they different?

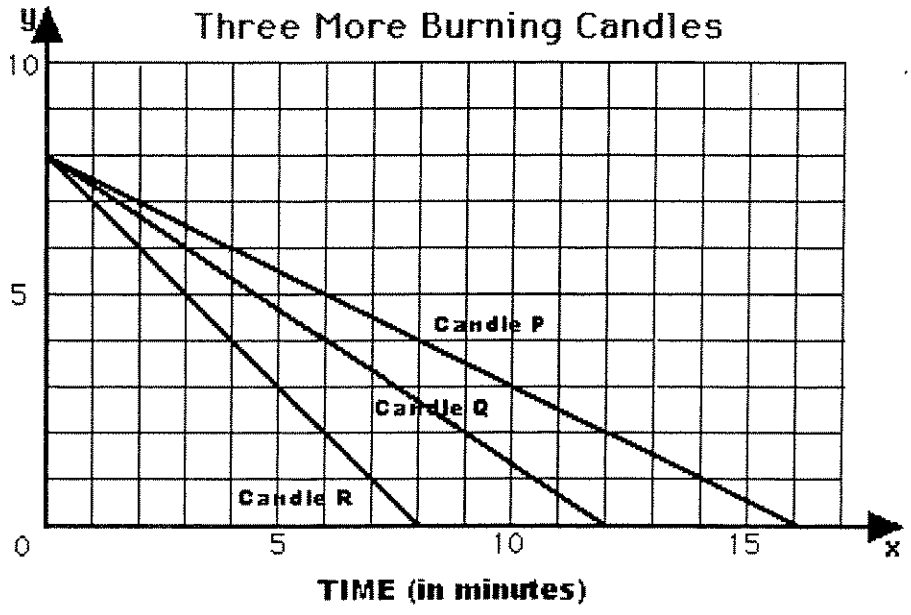
Same slopes, different initial heights

g. Write an equation for a fourth candle, Candle D, whose graph is parallel to the graphs of Candles A, B, and C.

h. Sketch the graph of your equation for Candle D on the grid with the graphs for Candles A, B, and C.

i. Describe the candle (Candle D) which you've represented by the graph and equation in parts g and h.

5. **Burning Candles, Part 3.**



a. What was the starting height of each candle?

all the same — 8 inches

b. At what rate did each candle burn? How do you know?

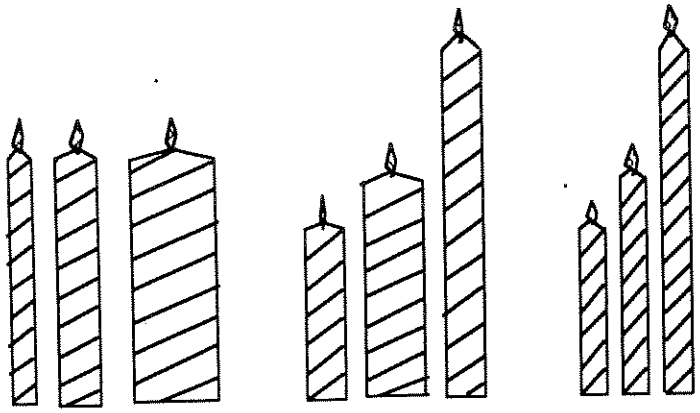
P 1 in : 2 minutes
Q 2 in : 3 minutes
R 1 in : 1 minute

c. What is the slope of each line?

P slope = $-\frac{1}{2}$
Q slope = $-\frac{2}{3}$
R slope = -1

5 cont.

d. Which of the following groups of candles could be represented by the graph and why? Which candle is Candle P? Candle Q? Candle R?



e. Write an equation to represent each candle's burning. (Make a table of data for each candle first, if it will help.)

P $H = 8 - \frac{1}{2}T$

Q $H = 8 - \frac{2}{3}T$

R $H = 8 - T$

f. How are the three equations alike? How are they different?

same initial heights
different slopes

g. Write an equation for a fourth candle, Candle S, whose graph is will intersect the point (0,8) like the other 3 graphs.

h. Sketch the graph of your equation for Candle S on the grid with the graphs for Candles P, Q, and R.

i. Describe or draw the candle (Candle S) which you've represented by the graph and equation in parts g and h.