Ch. 6.

Name Date

Division properties of exponents

Working with Powers, Exponents, and Polynomials

When dividing exponents, it is important to remember the following properties:

1. When dividing powers that have the same base, subtract the exponents.

   For example, $\frac{x^4}{x^2} = x^{4-2} = x^2$, where x cannot be equal to 0.

2. When finding a power of a quotient, find the power of the numerator and the power of the denominator and divide.

   For example, $(\frac{x}{y})^3 = \frac{x^3}{y^3}$, where y cannot be equal to 0.

Simplify $\frac{6^8}{6^6}$

$= 6^{8-6}$

$= 6^2 = 36$

Simplify $(\frac{3}{4})^{-2}$

$= \frac{3^{-2}}{4^{-2}}$

$= \frac{4^2}{3^2} = \frac{16}{9}$

1. Explain what you do with the exponents when dividing powers that have the same base.

Evaluate each expression.

2. $\frac{5^6}{5^3}$

3. $\frac{(-3)^2}{3^2}$

4. $\frac{3^2}{3^{-4}}$

5. $\frac{5^4 \cdot 5}{5^7}$

6. $(\frac{3}{2})^3$

7. $\frac{7^3}{7}$

8. $\frac{4^8}{4^8}$

9. $\frac{6^4 \cdot 6^3}{6^5}$

10. $(\frac{4}{5})^2$

11. $(\frac{-3}{4})^{-2}$

Simplify each expression.

12. $(\frac{3}{x})^3$

13. $x^5 \cdot \frac{1}{x^7}$

14. $\frac{18x^4y^2}{-6x^2y^4} \cdot \frac{-3x^2y^2}{-y}$

15. $\frac{x^3}{x^5}$

16. $\frac{4x^4y^4}{4x^2y^2} \cdot \frac{4x^2y^4}{2xy}$

17. $\frac{7x^{-3}y^3}{x^2y^{-3}} \cdot \frac{(2x^3y)^{-2}}{x^2y^2}$

18. $x^4 \cdot \frac{1}{x^2}$

19. $\frac{6x^2y^4}{3y^2} \cdot \frac{7x^2y^{-4}}{x^4}$

20. $\frac{8x^{-2}y^4}{x^3y^{-3}} \cdot \frac{(4xy^2)^{-1}}{x^{-2}y^{-2}}$

Name ____

Incredible Irony

Believe it or not, the people of Washington, D.C., did not always have the right to vote for the position of President of the United States. How many Presidents had the United States had before these people were allowed to vote?

To find out, simplify each term. Match your answers to those given. Write the letter corresponding to the solution above each problem number at the bottom of the page to spell out the answer.

_____ **1.** $\frac{x^2y}{xy}$

_____ **2.** $\frac{{}^-2x^2y}{18xy}$

_____ **3.** $\frac{10x^2y}{10x^2y}$

_____ **4.** $\frac{13xy}{26x^2y}$

_____ **5.** $\frac{(9y)^3}{9y}$

_____ **6.** $\frac{{}^-18xy^2}{6x^2y^3}$

_____ **7.** $\frac{{}^-12x^2y}{9xy^4}$

_____ **8.** $\frac{({}^-3xy)(6x^2y^4)}{9x^3y^2}$

_____ **9.** $\frac{(3x^2)^3(2xy)}{6xy}$

R. $\frac{1}{2x}$

S. $\frac{{}^-4x}{3y^3}$

T. x

Y. $\frac{{}^-3}{xy}$

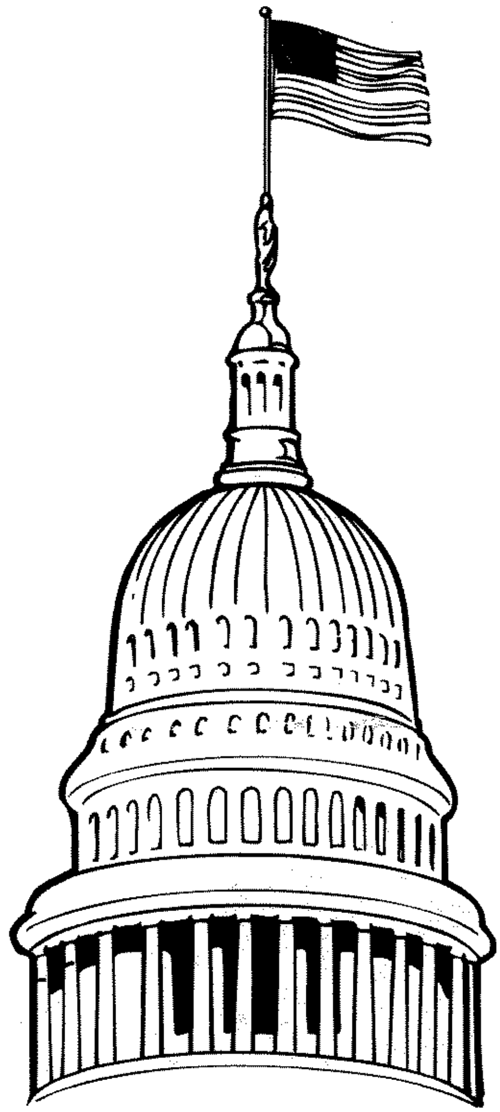
X. $9x^6$

H. $\frac{{}^-x}{9}$

I. ${}^-2y^3$

T. $81y^2$

I. 1



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1 2 3 4 5 6 7 8 9

Name Date

Simplifying rational expressions

A fraction whose numerator and denominator are polynomials is called a rational expression. To simplify rational expressions, follow two steps.

1. Factor the numerator and the denominator.
2. Divide any factors common to both the numerator and the denominator.

It is important to remember that an expression is undefined for a value if the value of a variable causes the denominator in the expression to be 0.

Find the values for which $\frac{x-4}{x^2+3x+2}$ is undefined.

$x^2 + 3x + 2$	First, set the denominator equal to 0.
$(x+2)(x+1)$	Factor denominator.
$x + 2 = 0$ or $x + 1 = 0$	Set each factor equal to 0.
$x = -2, x = -1$	Solve.

Thus, $\frac{x-4}{x^2+3x+2}$ is undefined for $x = -2$ or $x = -1$.

Simplify $\frac{3x-12}{x^2-x+12}$

$\frac{3(x-4)}{(x+3)(x-4)}$	Factor both the numerator and denominator. Cancel out what is common to both.
$\frac{3}{x+3}$	Simplified expression

Find the value(s) of x that would make the rational expression undefined.

1. $\frac{x-4}{3x+9}$
2. $\frac{x}{2x+6}$
3. $\frac{x}{x^2+8x+7}$
4. $\frac{5x+6}{x^2+3x-18}$
5. $\frac{-14}{6x-12}$
6. $\frac{x-15}{4x-16}$
7. $\frac{x-4}{x^2-5x-6}$
8. $\frac{x-9}{x^2-10x+25}$

Simplify each expression.

9. $\frac{16x}{24}$
10. $\frac{15x}{60x^4}$
11. $\frac{x-7}{x^2-7x}$
12. $\frac{x^2+7x}{x^2+5x-14}$
13. $\frac{-36x}{84x}$
14. $\frac{2x-12}{x-6}$
15. $\frac{x^2-36}{x^2+4x-12}$
16. $\frac{6x-24}{x^2-4x}$

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Simplifying rational expressions to convenient form

Calculating Rational Expressions

A rational expression is said to be simplified in convenient form if:

1. The expression is written in descending order of exponents.
2. The first coefficient, or leading coefficient, is positive.

Write $-3x^2 + x + 9 - x^3$ in convenient form.

$-x^3 - 3x^2 + x + 9$	Write in descending order of exponents.
$-1(x^3 + 3x^2 - x - 9)$	Factor out -1 to obtain a positive first coefficient.
$-1(x^3 + 3x^2 - x - 9)$	Convenient form

Simplify

$\frac{-2 + 5x - 3x^2}{-4 + 8x - 3x^2}$	
$\frac{-3x^2 + 5x - 2}{-3x^2 + 8x - 4}$	Rewrite the numerator and denominator in descending order of exponents.
$\frac{-1(3x^2 - 5x + 2)}{-1(3x^2 - 8x + 4)}$	Factor out -1 to obtain a positive first coefficient.
$\frac{\cancel{-1(3x-2)}(x - 1)}{\cancel{-1(3x-2)}(x - 2)}$	Factor the numerator and denominator. Cancel out what is common to both.
$\frac{x - 1}{x - 2}$	Simplified convenient form

Change each expression to convenient form.

1. $-6x - 5$
2. $36 - x^2$
3. $52 + y - x^2$
4. $11 - 2x + 2x^2$
5. $-x^2 + 10$
6. $-x^2 + 5x - 7$
7. $-x^2 + 9x - 4$
8. $-9x - 7$

Simplify each expression. Write each answer in convenient form.

9. $\frac{10 - x}{5x - 50}$
10. $\frac{x - 1}{1 - x^2}$
11. $\frac{18x^2 - 3x}{-6x^2 + 7x - 1}$
12. $\frac{2 - x}{x^2 + 4x - 12}$
13. $\frac{x^2 - 5x + 6}{3 - x}$
14. $\frac{9 - x}{x^2 - 8x - 9}$
15. $\frac{4x^3 - 16x^2 + 12x}{12x - 4x^2}$
16. $\frac{-2x^2 - x + 6}{x^2 - 2x - 8}$

Evaluate each expression for its given x-value.

17. $-x^2 + 2x - 16$ for $x = 2$
18. $\frac{-6 - x}{x^2 + 3x - 1}$ for $x = -1$

Name Date

Multiplying rational expressions

Calculating Rational Expressions

When multiplying rational expressions, follow these three steps.

1. Multiply the numerators.
2. Multiply the denominators.
3. Write the new fraction in reduced form.

Multiply $\frac{7x^3}{4y^2} \bullet \frac{2y^3}{8x^5}$

$7x^3 \bullet 2y^3 = 14x^3y^3$ Multiply the numerators.

$14y^2 \bullet 8x^5 = 32x^5y^2$ Multiply the denominators.

$\frac{4x^3y^3}{32x^5y^2}$ Write the new fraction.

$\frac{7y}{16x^2}$ Write the new fraction in reduced form.

Note: It is faster to cancel factors in the numerator and the denominator before multiplying.

Multiply $\frac{x^2 - 2x - 8}{x^2 - 2x - 3} \bullet \frac{x - 3}{x + 2}$

$\frac{(x + 2)(x - 4)}{(x - 3)(x + 1)} \bullet \frac{x - 3}{x + 2}$ Factor the numerators and denominators.

$\frac{\cancel{(x + 2)}(x - 4)}{\cancel{(x - 3)}(x + 1)} \bullet \frac{\cancel{x - 3}}{\cancel{x + 2}}$ Cancel out the common factors.

$\frac{x - 4}{x + 1} \bullet 1$ Rewrite the remaining factors. Multiply.

$\frac{x - 4}{x + 1}$ Final product

Multiply.

1. $\frac{8}{11} \bullet \frac{5}{9}$

2. $\frac{x-8}{x+2} \bullet x-8$

3. $\frac{4x^4y^6}{x^2-9} \bullet \frac{6-2x}{8x^6y^4}$

4. $\frac{3}{10} \bullet \frac{-9}{13}$

5. $\frac{x+2}{22} \bullet \frac{11}{x+2}$

6. $(x+4) \bullet \frac{3x-6}{x^2+2x-8}$

7. $\frac{7a}{5} \bullet \frac{a^3}{4}$

8. $\frac{-6x}{6x-12} \bullet \frac{3x-6}{12x^3}$

9. $\frac{16x^7y}{7-x} \bullet \frac{3x-21}{32x^4y^6}$

10. $\frac{6x}{17y^2} \bullet \frac{4y^8}{3x^9}$

11. $\frac{45}{6x+8y} \bullet \frac{9x+12y}{9}$

12. $\frac{x^2-y^2}{3x^2-21xy+30y^2} \bullet \frac{75y^2-3x^2}{x^2-2xy+y^2}$

13. $\frac{-5x^5}{9y^2} \bullet \frac{12y^{10}}{10x^7}$

14. $\frac{4x-24}{4x^2+18x-10} \bullet (2x-1)$

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Dividing rational expressions

To divide one rational expression by another, multiply the first by the reciprocal of the second. Be sure not to confuse the operation. Invert the divisor (the second fraction) only.

Divide $\frac{15x^6}{5y^5} \div \frac{3x^4}{5y^4}$

$\frac{^5\cancel{15}\cancel{x^6}^2}{\cancel{5}y^{\cancel{5}}} \cdot \frac{\cancel{5y^4}}{\cancel{3x^4}}$ Change to multiplication. Cancel common factors.

$\frac{5x^2}{y} \cdot 1$ Multiply.

$\frac{5x^2}{y}$ Final product

Divide $\frac{x^2+9x+20}{x^2-5x+6} \div \frac{x+4}{x-2}$

$\frac{x^2+9x+20}{x^2-5x+6} \cdot \frac{x-2}{x+4}$ Change to multiplication.

$\frac{\cancel{(x+4)}(x+5)}{(x-3)\cancel{(x-2)}} \cdot \frac{\cancel{x-2}}{\cancel{x+4}}$ Factor the numerator and denominator and cancel common factors.

$\frac{x+5}{x-3} \cdot 1$ Multiply.

$\frac{x+5}{x-3}$ Final product

Give the reciprocal of each rational expression.

1. 10
2. $x + 12$
3. $\frac{3}{4}$
4. $x^2 - 7$
5. $\frac{1}{x-4}$
6. $\frac{3x^2}{x^2-5x+8}$
7. 0
8. $\frac{4x^2-3x+7}{5x-12}$

Divide.

9. $\frac{8}{9} \div \frac{1}{9}$
10. $\frac{15x^7y^6}{4x-8x^2} \div \frac{24x^6y^8}{8x-4}$
11. $\frac{4x^2-25y^2}{2x^2y+5xy^2} \div \frac{6x^2-15xy}{9x^2y^2}$
12. $\frac{x^2}{y^3} \div \frac{y^3}{x^5}$
13. $\frac{x^2-2x-15}{5x^2+15x} \div x^2-6x+5$
14. $\frac{x^2-x-12}{x^2+x-6} \div \frac{x^2-2x-8}{x^2-x-2}$
15. $\frac{3xy^2}{-5z^5} \div \frac{6xz^2}{10y^3}$
16. $\frac{x-8}{4} \div \frac{x+8}{8}$
17. $\frac{x^2-81}{x^2-3x-54} \div \frac{2x+18}{6x+36}$
18. $\frac{4x-12}{12} \div \frac{x^2-9}{24}$
19. $\frac{x^2-25}{x-3} \div \frac{5x+25}{2x-6}$
20. $\frac{2x^2+5x+3}{x^2+9x+14} \div \frac{2x^2-3x-9}{x^2+6x-7}$

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Adding and subtracting with like denominators

Calculating Rational Expressions

When adding and subtracting rational expressions with like denominators, add/subtract the numerators and write the result as a fraction with the common denominator.

Add $\frac{8x}{4} + \frac{5x}{4}$

$8x + 5x$ Add the numerators.

$\frac{13x}{4}$ Write as a fraction with the common denominator.

Subtract $\frac{5x}{x-2} - \frac{4x}{x-2}$

$5x - 4x$ Subtract the numerators.

$\frac{x}{x-2}$ Write as a fraction with the common denominator.

Note: Before adding and subtracting rational expressions, the expressions need to be in convenient form.

Add $\frac{x^2}{x^2+2x-8} + \frac{-3x+10}{-x^2-2x+8}$

$\frac{x^2}{x^2+2x-8} - \frac{-3x+10}{+(x^2+2x-8)}$ Write $-x^2 - 2x + 8$ in convenient form. $-1(x^2 + 2x - 8)$
-1 means addition changes to subtraction.
Factor denominators only.

$\frac{x^2+3x+10}{(x+4)(x-2)} = \frac{(x+5)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$ Subtract numerators. Then factor if possible. Cancel common factors.

$\frac{x+5}{x+4}$ Final answer

Note: When subtracting an expression, remember to distribute the negative sign to each term in the expression.

Add or subtract each expression.

1. $\frac{9x}{4} + \frac{8x}{4}$

2. $\frac{2x}{5} - \frac{x-1}{5}$

3. $\frac{x}{2x-6} - \frac{3}{2x-6}$

4. $\frac{5x^2}{6} + \frac{8x^2}{6}$

5. $\frac{4x}{2x} - \frac{2x+2}{2x}$

6. $\frac{6x}{x^2+8x} + \frac{48}{x^2+8x}$

7. $\frac{x}{x-5} + \frac{5}{5-x}$

8. $\frac{3}{x} + \frac{2}{x} - \frac{1}{x}$

9. $\frac{3x}{x^2+2x-15} + \frac{15}{x^2+2x-15}$

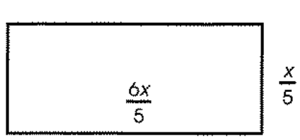
10. $\frac{x+12}{x} - \frac{5}{x}$

11. $\frac{8x}{x-6} + \frac{3x}{6-x}$

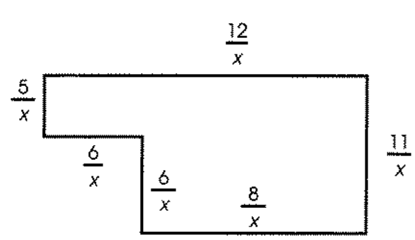
12. $\frac{2x^2}{x-1} + \frac{2}{1-x}$

Find the perimeter of each figure.

13.



14.



Name Date

Adding and subtracting with unlike denominators

Calculating Rational Expressions

To add and subtract expressions with unlike denominators, first find the LCD (least common denominator). The LCD is found by factoring each denominator into prime factors and identifying the most number of times each factor must occur.

$\frac{3}{4x} + \frac{7}{16x}$ Find the LCD of $4x$ and $16x$. (The LCD is the least common multiple of $4x$ and 16.)

$4x = 2 \bullet 2 \bullet x$

$16x = 2 \bullet 2 \bullet 2 \bullet 2 \bullet x$

Since 2 most occurs 4 times and x most occurs once, the LCD is $2 \bullet 2 \bullet 2 \bullet 2 \bullet x = 16x$. Once the LCD is found, multiply each term by the factors in the LCD it is missing. Be sure to multiply the numerator and the denominator of each term by the same factors.

Add $\frac{1}{3x^3} + \frac{7}{2x^2} + \frac{3}{6x}$

$3x^3 = 3 \bullet x \bullet x \bullet x$ Find the prime factors of each denominator.

$2x^2 = 2 \bullet x \bullet x$

$6x = 2 \bullet 3 \bullet x$

$2 \bullet 3 \bullet x \bullet x \bullet x = 6x^3$ Common denominator

$\frac{2}{2}\left(\frac{1}{3x^3}\right) + \frac{3x}{3x}\left(\frac{7}{2x^2}\right) + \frac{x^2}{x^2}\left(\frac{3}{6x}\right)$ Multiply the numerator and denominator of each term by the LCD factors the denominator is missing.

$\frac{2 + 21x + 3x^2}{6x^3}$ Rewrite with the common denominator.

$\frac{3x^2 + 21x + 2}{6x^3}$ Simplify.

Find the LCD of each set of fractions.

1. $\frac{5}{2} + \frac{3}{5} - \frac{3}{4}$

2. $\frac{4x}{3x^3} + \frac{x}{x^2} - \frac{6}{8x}$

3. $\frac{9x}{4xy} - \frac{12x}{6x}$

4. $\frac{4x-5}{3x^2} - \frac{5x-1}{4x^3}$

5. $\frac{4x+3}{3} - \frac{3x+2}{7}$

6. $\frac{1}{3x^4} - \frac{4}{x^6} + \frac{3}{6x^2}$

Add or subtract each expression. Each answer should be in the simplest form.

7. $\frac{6x}{15} + \frac{2x}{5}$

8. $\frac{7}{24x} + \frac{3x-4}{6x} + \frac{x+3}{4x}$

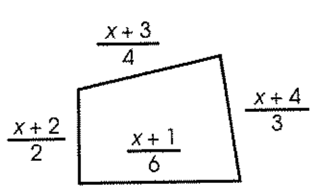
9. $\frac{7}{5x^2} - \frac{1}{10x} + \frac{9}{4x^4}$

10. $\frac{3x}{4} - \frac{x}{2}$

11. $\frac{1}{x^3} + \frac{2}{x} - \frac{3}{4x^2}$

12. $\frac{3x+8}{6x} + \frac{3x-2}{4x}$

13. Find the perimeter of the figure.



Name Date

Adding and subtracting with polynomial denominators

Calculating Rational Expressions

Adding and subtracting rational expressions will often involve polynomial denominators. The first step to take when solving such problems is to find the LCD. Then replace the rational expressions with equivalent expressions that have the LCD as the denominator.

1. Add $\frac{5}{x-2} + \frac{7}{x-6}$

$(x-2)(x-6)$ — Since $x-2$ and $x-6$ are both irreducible, the LCD is $(x-2)(x-6)$.

$\frac{(x-6)}{(x-6)} \bullet \frac{5}{x-2} + \frac{(x-2)}{(x-2)} \bullet \frac{7}{x-6}$ — Multiply the numerator and the denominator of each term by its missing factor.

$\frac{5x-30+7x-14}{(x-2)(x-6)}$ — Rewrite with the LCD.

$\frac{12x-44}{(x-2)(x-6)} = \frac{4(3x-11)}{(x-2)(x-6)}$ — Add and find the final result.

2. Subtract $(x+2) - \frac{(x-1)}{2x+1}$

1 and $2x+1$; LCD = $2x+1$ — Find the LCD.

$\frac{2x+1}{2x+1}(x+2) - \frac{(x-1)}{2x+1}$ — Multiply the numerator and the denominator of each term by its missing factor.

$\frac{2x^2+5x+2-x+1}{2x+1}$ — Rewrite with the LCD.

$\frac{2x^2+4x+3}{2x+1}$ — Add like terms and find the final result.

Find the LCD of each set of fractions.

1. $\frac{4}{x-3} + \frac{-3}{x+2}$ **2.** $\frac{4}{x-1} + \frac{3}{x+6}$ **3.** $12 + \frac{3}{x}$ **4.** $(x+4) + \frac{8}{x-3}$

5. $\frac{6}{2x+3} - \frac{7}{x-4}$ **6.** $3 - \frac{5}{x}$ **7.** $\frac{5}{x^2} + 1$ **8.** $(x-1) - \frac{3x}{x+2}$

Add or subtract each expression.

9. $\frac{4}{x-3} + \frac{2}{x}$ **10.** $\frac{3x}{x^2-36} - \frac{18}{6-x}$ **11.** $(x+4) + \frac{x-5}{4x-3}$

12. $6 - \frac{1}{x+6}$ **13.** $\frac{x}{x-4} - \frac{10}{x+4}$ **14.** $\frac{5}{x-7} - \frac{4}{x^2-5x-14}$

WHY ISN'T A SNOWMAN VERY SMART?

Express each difference below in simplest form. Find your answer and notice the letter next to it. Write this letter in each box containing the number of that exercise.

1. $\frac{8}{x^2-4} - \frac{3}{x-2}$
2. $\frac{9}{x^2-2x-15} - \frac{2}{x+3}$
3. $\frac{7x}{x^2-9x+14} - \frac{4}{x-7}$
4. $\frac{3}{x-4} - \frac{x-9}{x^2-16}$
5. $\frac{5}{x+5} - \frac{2x+5}{x^2+9x+20}$
6. $\frac{3}{d-7} - \frac{2}{3d+1}$
7. $\frac{8}{5d+4} - \frac{1}{2d-3}$
8. $\frac{d+2}{4d-1} - \frac{7}{d+5}$
9. $\frac{d^2+3}{d^2-2d} - \frac{d-4}{d}$
10. $\frac{d^2-11}{d^2-7d+12} - \frac{d+1}{d-4}$

Answers:

(L) $\frac{3x}{x+5}$ (A) $\frac{-2x+19}{(x+3)(x-5)}$

(I) $\frac{3}{x+4}$ (U) $\frac{2x+3}{(x-2)(x-7)}$

(O) $\frac{-3x+2}{(x+2)(x-2)}$ (W) $\frac{2x+21}{(x+4)(x-4)}$

(E) $\frac{3x+8}{(x-2)(x-7)}$ (C) $\frac{7x+11}{(x+3)(x-5)}$

Answers:

(Y) $\frac{3d+8}{d(d-2)}$ (P) $\frac{8d-15}{(5d+4)(2d-3)}$

(S) $\frac{2}{d-3}$ (H) $\frac{7d+17}{(d-7)(3d+1)}$

(N) $\frac{d^2-21d+17}{(4d-1)(d+5)}$ (T) $\frac{d^2-18d+4}{(4d-1)(d+5)}$

(R) $\frac{6d-5}{d(d-2)}$ (B) $\frac{11d-28}{(5d+4)(2d-3)}$

6	3	6	2	10	10	8	1	4	7	9	2	5	8	10

OBJECTIVE 1–e: To subtract algebraic fractions.

Name Date

Ratios and proportions

Applying Rational Equations and Proportions

A ratio is a comparison of two numbers by division. For example, $\frac{3}{x}$ is a ratio.
When two ratios are set equal to each other, this is called a proportion. For example, $\frac{3}{x} = \frac{2}{3}$ is a proportion. There are two properties of proportions to remember.

1. If two ratios are equal, then their reciprocals are equal.
   if $\frac{6}{2} = \frac{12}{4}$, then $\frac{2}{6} = \frac{4}{12}$.
2. The product of the extremes equals the product of the means.
   Example, if $\frac{3}{x} = \frac{1}{4}$, then $3(4) = x(1)$.
   The extremes are 3 and 4, and the means are x and 1.

Problems that involve proportions often involve a variable. When solving for the variable, the proportion is actually being solved.

Solve $\frac{x}{(x+6)} = \frac{3}{x}$

extremes: x and x; means: $x + 6$ and 3	First, identify the extremes and the means.
$3(x + 6) = x(x)$	Cross multiply.
$3x + 18 = x^2$	Simplify.
$x^2 - 3x - 18 = 0$	Write in standard form.
$(x - 6)(x + 3) = 0$	Factor.
$x = 6$ and $x = -3$	Solve for x.

Thus, the solution of this proportion is 6 and -3.

Identify the extremes and the means in each proportion.

1. $\frac{4}{15} = \frac{3x}{10}$ **2.** $\frac{x}{4} = \frac{8}{3}$ **3.** $\frac{x}{2} = \frac{3}{x+2}$ **4.** $\frac{x+4}{6} = \frac{x+7}{7}$

5. $\frac{5}{6} = \frac{45}{n}$ **6.** $\frac{11}{x} = \frac{44}{121}$ **7.** $\frac{x}{3} = \frac{4}{x-5}$ **8.** $\frac{4}{x+8} = \frac{3}{x}$

Solve each proportion.

9. $\frac{x}{4} = \frac{21}{7}$ **10.** $\frac{9}{3} = \frac{6}{x}$ **11.** $\frac{x-3}{4} = \frac{x-4}{3}$ **12.** $\frac{x}{4} = \frac{8}{2x}$

13. $\frac{12}{x} = 3$ **14.** $\frac{x}{5} = \frac{4}{x-1}$ **15.** $\frac{x}{2} = \frac{18}{x+5}$ **16.** $\frac{13}{x} = \frac{x}{13}$

17. What is the difference between a ratio and a proportion? Give a definition of both.

Name Date

Rational equations

Applying Rational Equations and Proportions

A rational equation contains one or more rational expressions separated with an equal sign. To solve a rational equation, follow these three steps.

1. Multiply each side of the equation by the least common denominator of each fraction in the equation.
2. Simplify each term in the equation.
3. Solve for x in the resulting polynomial equation using standard techniques.

Solve $\frac{5}{2x} + \frac{5}{x} = \frac{1}{3}$

$6x(\frac{5}{2x} + \frac{5}{x}) = \frac{1}{3}(6x)$	Multiply each side of the equation by LCD, $6x$.
$\frac{30x}{2x} + \frac{30x}{x} = \frac{6x}{3}$	Multiply each term by $6x$.
$15 + 30 = 2x$	Simplify.
$45 = 2x$	Divide by 2.
$x = 22\frac{1}{2}$	Solve for x.

Note: When each side of an equation is a single fraction, use cross multiplying to solve the rational equation.

Solve $\frac{6}{x + 4} = \frac{x}{2}$

$x(x + 4) = 12$	Cross multiply.
$x^2 + 4x = 12$	Simplify.
$x^2 + 4x - 12 = 0$	Write the equation in standard form.
$(x + 6)(x - 2) = 0$	Factor.
$x = -6, x = 2$	Solve for x.

Thus, the solutions are -6 and 2.

Find the greatest common factor.

1. 4, $12x$, 6 **2.** x, $5x^3$, x^2 **3.** x^4, $-6x^2$, x^3 **4.** $4x$, $6x$, $8x$

Solve each equation by cross multiplying.

5. $\frac{5}{x} = \frac{10}{3}$ **6.** $\frac{1}{3x+3} = \frac{1}{x+4}$ **7.** $\frac{3}{x+9} = \frac{x}{x-3}$ **8.** $\frac{2}{x+2} = \frac{3}{x+3}$

Solve each equation.

9. $\frac{1}{3} + \frac{2}{x} = \frac{5}{6}$ **10.** $x + 10 = \frac{-25}{x}$ **11.** $\frac{2}{(x-3)^2} = 1 - \frac{1}{x-3}$

12. $\frac{36}{x} = -12 - x$ **13.** $\frac{1}{x-4} + \frac{1}{x+4} = \frac{16}{x^2-16}$ **14.** $\frac{7}{3x-12} - \frac{1}{x-4} = \frac{4}{3}$

Name Date

Problem solving with work problems

Applying Rational Equations and Proportions

A formula to remember when solving work problems is:

Time • rate = part of work completed: $w = r \cdot t$

Joe and Bob are going to paint Bob's house. Joe takes 5 days to paint a house, and Bob takes 7 days to paint the very same house. If they work together, how long will it take them to paint Bob's house?

First, set up the plan.

Let x = the number of days Joe and Bob work together.

Joe's part of the job done in one day = $\frac{1}{5}$

Bob's part of the job done in one day = $\frac{1}{7}$

Joe's part of the job completed = $\frac{1}{5}x$ or $\frac{x}{5}$

Bob's part of the job completed = $\frac{1}{7}x$ or $\frac{x}{7}$

Joe's part + Bob's part = whole job

Now, plug the values into the equation and solve for x.

$\frac{x}{5} + \frac{x}{7} = 1$	
$35(\frac{x}{5} + \frac{x}{7}) = 1(35)$	Multiply by the common denominator.
$7x + 5x = 35$	Simplify.
$12x = 35$	Divide by 12.
$x = 2\frac{11}{12}$ days	Solve for x.

Thus, if Joe and Bob work together, they can have Bob's house painted in $2\frac{11}{12}$ days.

Solve each problem.

1. Stacey can type a document in 6 hours. What part can she type in 2 hours? in n hours?

2. John can bike to his grandma's house in 1 hour. What part of the route can he bike in 45 minutes?

3. Jane takes 12 hours to clean her dad's office building. Lisa has cleaned it in 14 hours. How long would it take the girls if they cleaned it together?

4. It takes Matt 10 hours to wallpaper a bathroom. Dan has done the same bathroom in 8 hours. How long would it take if the boys wallpapered it together?

5. Ashley and Will can paint their bedroom in 5 hours if they work together. If it takes Ashley 9 hours to paint it alone, how long would it take Will to paint it alone?

6. It takes Heather 22 hours to wallpaper two rooms. She and Jill can do it together in 16 hours. How long would it take Jill to wallpaper alone?

EXTRA PRACTICE 18
Solving Rational Equations
Use after Section 7.6

Name______________________________

Example: Solve. $\frac{5}{x+2} = \frac{3}{x}$

The LCM is $x(x+2)$

$$x(x+2)\left(\frac{5}{x+2}\right) = x(x+2)\left(\frac{3}{x}\right)$$
$$5x = 3(x+2)$$
$$5x = 3x + 6$$
$$2x = 6$$
$$x = 3$$

The solution is 3.

Check: $\frac{5}{x+2} = \frac{3}{x}$

$\frac{5}{3+2}$	$\frac{3}{3}$
$\frac{5}{5}$	1
1	1

Solve.

1. $\frac{4}{x-1} = \frac{5}{x}$ ____________________

2. $\frac{x-3}{x+2} = \frac{4}{5}$ ____________________

3. $\frac{5}{x} = \frac{4}{x} + \frac{1}{2}$ ____________________

4. $\frac{1}{3} - \frac{3}{4} = \frac{x}{12}$ ____________________

5. $\frac{4}{3x} + \frac{2}{x} = \frac{2}{3}$ ____________________

6. $\frac{8}{x-5} = \frac{2}{x+5}$ ____________________

7. $\frac{x-7}{x+3} = \frac{2x}{x+3}$ ____________________

8. $\frac{y-1}{4} - \frac{y+1}{10} = 1$ ____________________

EXTRA PRACTICE 19
Applications Using Rational Equations and Proportions
Use after Section 7.7 **Name**____________________________

Example: A number plus five times its reciprocal is -6. Find the number.

A number plus five times its reciprocal is -6.

$$x \quad + \quad 5 \quad \cdot \quad \frac{1}{x} \quad = -6$$

Solve: $x + 5 \cdot \frac{1}{x} = -6$

$$x\left(x + \frac{5}{x}\right) = x(-6) \quad \text{Multiplying by the LCD, } x, \text{ on both sides.}$$

$$x^2 + 5 = -6x$$

$$x^2 + 6x + 5 = 0$$

$$(x+5)(x+1) = 0$$

$$x = -5 \text{ or } x = -1$$

The values -5 and -1 check in the original problem. The solutions are -5 and -1.

Solve.

1. A number minus three times its reciprocal is 2. Find the number. ______________

2. The sum of a number and twice its reciprocal is 3. Find the number. ______________

3. It takes Carolyn 4 hr to type a final exam. It takes Elise 3 hr to do the same job. How long would it take them, working together, to do the typing? ______________

4. A swimming pool can be filled in 15 hr by pipe A alone and in 24 hr by pipe B alone. How long would it take to fill the pool if both pipes were working? ______________

5. One car travels 30 km/h faster than another. In the same time that one travels 200 km, the other goes 320 km. Find their speeds. ______________

6. The speed of a freight train is 16 mph slower than the speed of a passenger train. The freight train travels 420 miles in the same time that it takes the passenger train to travel 500 miles. Find the speed of each train. ______________

7. William walked 195 km in 12 days. At this rate, how far would he walk in 36 days?
____

8. The winner of an election for class president won by a vote of 8 to 5 with 992 votes. How many votes did the loser get? ______________

