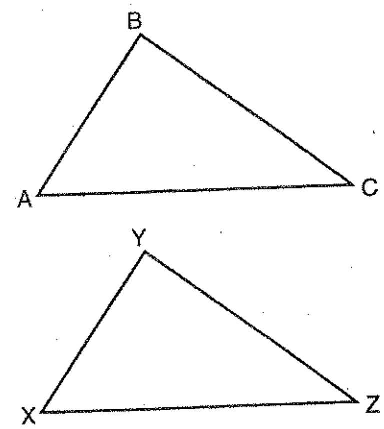
CONGRUENT TRIANGLES



Congruent triangles are commonly used in the construction of quilts, buildings, and bridges. Congruent triangles are also used to estimate inaccessible distances; for example, the width of a river or the distance across a canyon. In this lesson you will learn simple ways to find out if two triangles are congruent.

When you buy floor tiles, you get tiles that are all the same shape and size. One tile will fit right on top of another. In geometry, you would say one tile is *congruent* to another tile. Similarly, in the figure below, ΔABC and ΔXYZ are congruent. They have the same size and shape.

Imagine sliding one triangle over to fit on top of the other triangle. You would put point A on point X; point B on point Y; and point C on point Z. When the vertices are matched in this way, $\angle A$ and $\angle X$ are called corresponding angles, and $\overline{AB}$ and $\overline{XY}$ are called corresponding sides.

Corresponding angles and corresponding sides are often referred to as corresponding parts of the triangles. In other words, you could say Corresponding Parts of Congruent Triangles are Congruent (CPCTC). This statement is often referred to by the initials CPCTC.

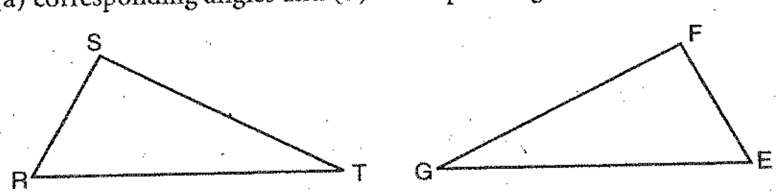
When ΔABC is congruent to ΔXYZ, you write $\Delta ABC \cong \Delta XYZ$. This means that all of the following are true:

$\angle A \cong \angle X$ $\quad \angle B \cong \angle Y$ $\quad \angle C \cong \angle Z$

$\overline{AB} \cong \overline{XY}$ $\quad \overline{BC} \cong \overline{YZ}$ $\quad \overline{AC} \cong \overline{XZ}$

Suppose instead of writing $\Delta ABC \cong \Delta XYZ$ you started to write $\Delta CAB \cong$ ______. Since you started with C to name the first triangle, you must start with the corresponding letter, Z, to name the second triangle. Corresponding parts are named in the same order. If you name the first triangle ΔCAB, then the second triangle must be named ΔZXY. In other words, $\Delta CAB \cong \Delta ZXY$.

Example: Name the (a) corresponding angles and (b) corresponding sides.



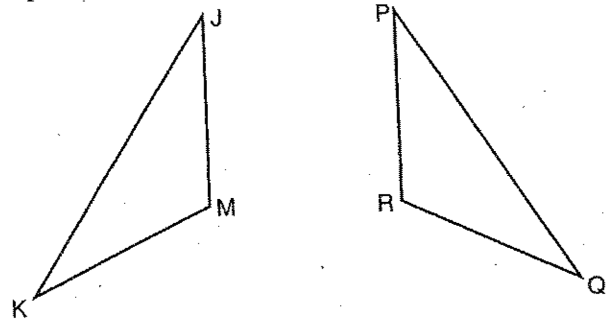
$\Delta RST \cong \Delta EFG$

Solution:

(a) corresponding angles: $\angle R$ and $\angle E$; $\angle S$ and $\angle F$; $\angle T$ and $\angle G$

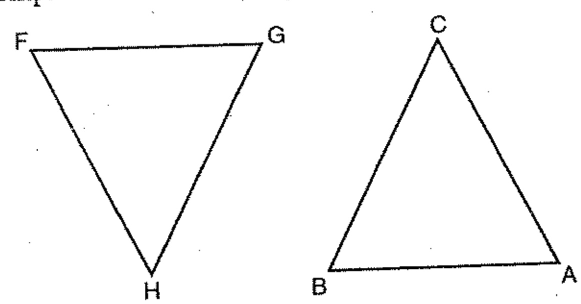
(b) corresponding sides: $\overline{RS}$ and $\overline{EF}$; $\overline{ST}$ and $\overline{FG}$; $\overline{RT}$ and $\overline{EG}$

For practice problems 1–6, complete each statement; given $\Delta JKM \cong \Delta PQR$



1. $\angle M$ corresponds to $\angle$______.
2. $\angle P$ corresponds to $\angle$______.
3. $\angle Q$ corresponds to $\angle$______.
4. $\overline{JK}$ corresponds to ______.
5. $\overline{RQ}$ corresponds to ______.
6. $\overline{PR}$ corresponds to ______.

For practice problems 7–10, complete each statement, given $\Delta FGH \cong \Delta ABC$.



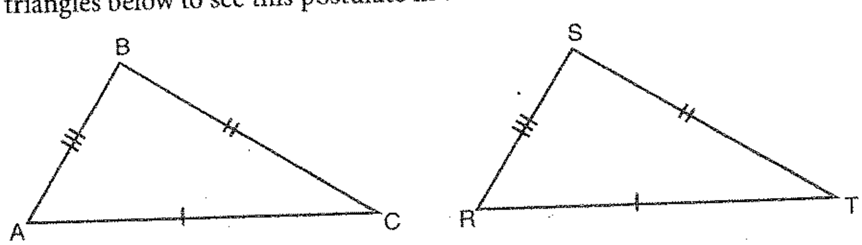
7. $\Delta FHG \cong \Delta$______.
8. $\Delta BAC \cong \Delta$______.
9. $\Delta HGF \cong \Delta$______.
10. $\Delta CAB \cong \Delta$______.

Side-Side-Side (SSS) Postulate

If you have three sticks that make a triangle and a friend has identical sticks, would it be possible for each of you to make different-looking triangles? No, it is impossible to do this. A postulate of geometry states this same idea. It is called the *Side-Side-Side Postulate.*

> *Side-Side-Side Postulate*: If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.

Take a look at the triangles below to see this postulate in action:



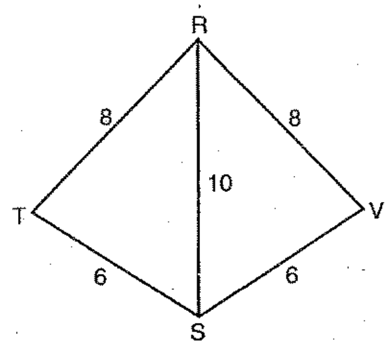
$\Delta ABC \cong \Delta RST$

The hatch marks on the triangles show which sides are congruent to which in the two triangles. For example, $\overline{AC}$ and $\overline{RT}$ both have one hatch mark, which shows that these two segments are congruent. $\overline{BC}$ is congruent to $\overline{ST}$, as shown by the two hatch marks, and $\overline{AB}$ and $\overline{RS}$ are congruent as shown by the three hatch marks.

Since the markings indicate that the three pairs of sides are congruent, you can conclude that the three pairs of angles are also congruent. From the definition of congruent triangles, it follows that all six parts of ΔABC are congruent to the corresponding parts of ΔRST.

PRACTICE

Use the figure below to answer questions 11–15.



11. $\overline{RS}$ corresponds to ________.

12. $\overline{TS}$ corresponds to ________.

13. $\overline{RV}$ corresponds to ________.

14. Is $\Delta RTS \cong \Delta RVS$?

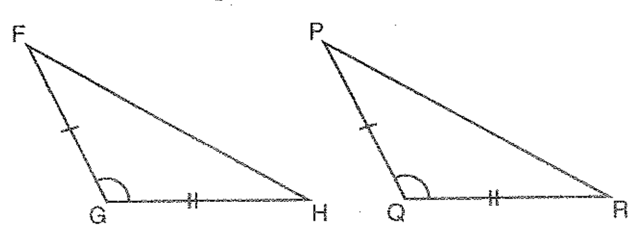
15. Is $\Delta RSV \cong \Delta RTS$?

Side-Angle-Side (SAS) Postulate

If you put two sticks together at a certain angle, there is only one way to finish forming a triangle. Would it be possible for a friend to form a different-looking triangle if she started with the same two lengths and the same angle? No, it would be impossible. Another postulate of geometry states this same idea; it is called the *Side-Angle-Side Postulate.*

Side-Angle-Side Postulate: If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent.

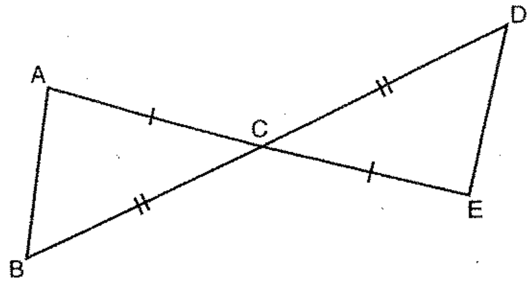
Look at the two triangles below to see an example of this postulate:



$\Delta FGH \cong \Delta PQR$

PRACTICE

Use the figure below to answer practice problems 16–20.



16. What kind of angles are ∠ACB and ∠ECD?

17. Is ∠ACB ≅ ∠ECD?

18. $\overline{CE}$ corresponds to ______.

19. $\overline{BC}$ corresponds to ______.

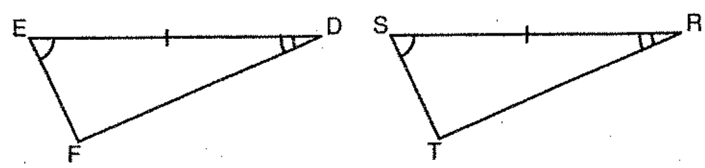
20. Is ΔACB ≅ ΔECD?

Angle-Side-Angle (ASA) Postulate

There is one more postulate that describes two congruent triangles. It involves two angles and a side between them. The side is called an included side.

> *Angle-Side-Angle Postulate:* If two angles and the included side of one triangle are congruent to corresponding parts of another triangle, the triangles are congruent.

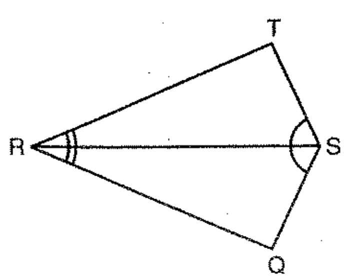
Take a look at the following two triangles:



ΔDEF ≅ ΔRST

PRACTICE

Use the figure below to answer practice problems 21–25.



21. ∠TRS corresponds to ∠______.

22. ∠QSR corresponds to ∠______.

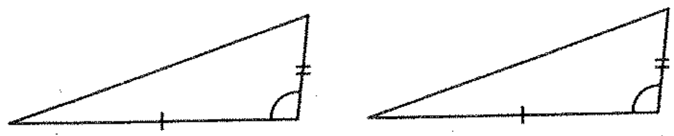
23. Is ΔRTS ≅ ΔRQS?

24. Is ΔSRT ≅ ΔSRQ?

25. Is ΔSRT ≅ ΔRQS?

State which postulate you would use to prove the two triangles congruent.

26.



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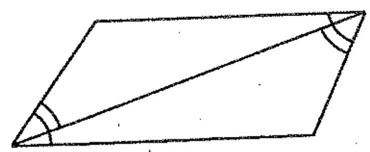

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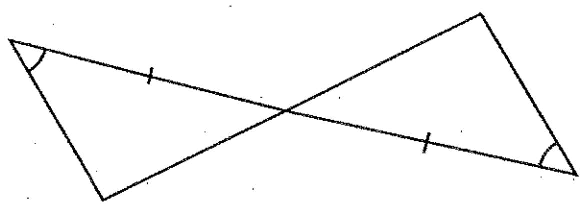

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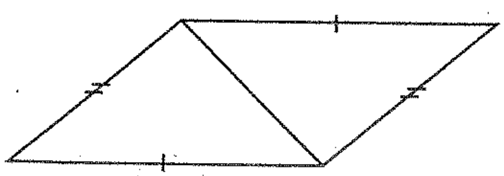

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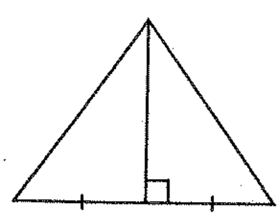


Answer Key
Congruent triangles

1. $\angle R$
2. $\angle J$
3. $\angle K$
4. $\overline{PQ}$
5. $\overline{MK}$
6. $\overline{JM}$
7. ΔACB
8. ΔGFH
9. ΔCBA
10. ΔHFG
11. $\overline{RS}$
12. $\overline{VS}$
13. $\overline{RT}$
14. yes
15. no
16. vertical angles
17. yes
18. $\overline{AC}$
19. $\overline{CD}$
20. yes
21. $\angle QRS$
22. $\angle TSR$
23. yes
24. yes
25. no
26. SAS
27. ASA
28. ASA
29. SSS
30. SAS

TRIANGLE SIMILARITY

You can prove that two figures are similar by using the definition of *similar*. In other words, two figures are similar if you can show that the following two statements are true:

(1) corresponding angles are congruent

(2) corresponding sides are in proportion

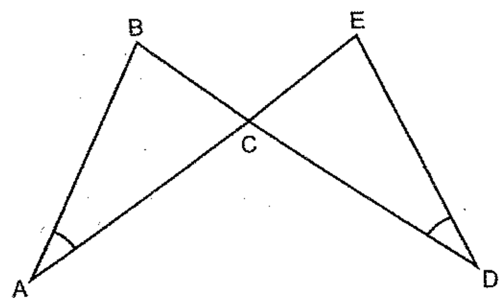
In addition to using the definition of similar, you can use three other methods for proving that two triangles are similar. The three methods are called the Angle-Angle Postulate, the Side-Side-Side Postulate, and the Side-Angle-Side Postulate.

If you know the measurements of two angles of a triangle can you find the measurement of the third angle? Yes, from Lesson 9, you know that the sum of the three angles of a triangle is 180°. Therefore, if two angles of one triangle are congruent to two angles of another triangle, then their third angles must also be congruent. This will help you to understand the next postulate. You should know that the symbol used for similarity is ~.

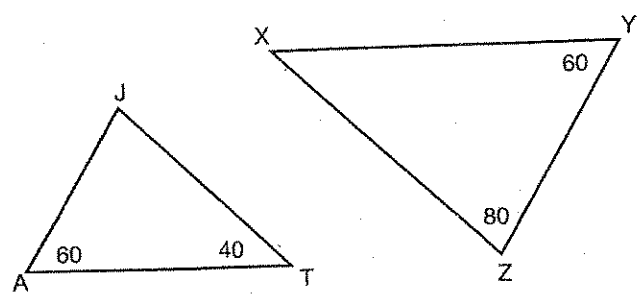
> *Angle-Angle Postulate (AA Postulate)*: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Examples: Are these triangles similar?

(a)



(b)



Solutions:

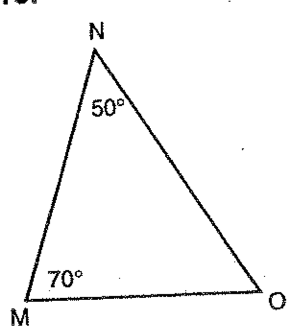
(a) $\angle A \cong \angle D$, given
$\angle BCA \cong \angle ECD$, vertical $\angle$'s are $\cong$
$\Delta ABC \sim \Delta DEC$, AA Postulate

(b) $\angle J = 180 - (60 + 40)$
$\angle J = 80$
$\Delta AJT \sim \Delta YZX$, AA Postulate

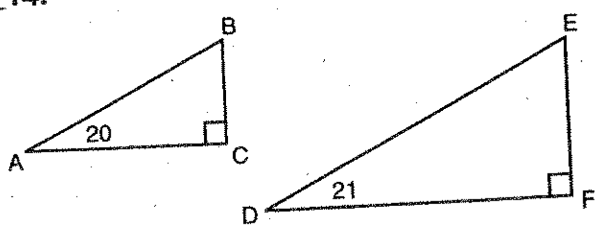
ACTICE

State whether the triangles are similar.

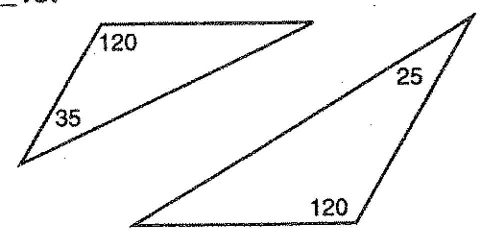
______ **13.**



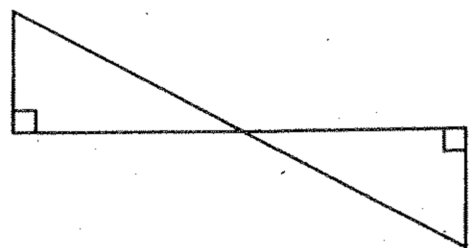
______ **14.**



______ **15.**



______ **16.**

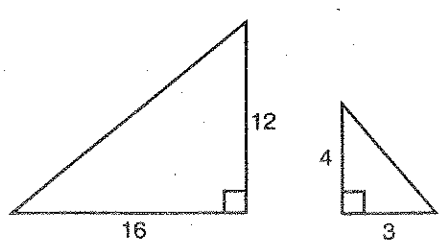


Here are two more postulates you can use to prove that two triangles are similar:

> *Side-Side-Side Postulate (SSS Postulate):* If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.
> *Side-Angle-Side Postulate (SAS Postulate):* If the lengths of two pairs of corresponding sides of two triangles are proportional and the corresponding included angles are congruent, then the triangles are similar.

Examples: Which postulate, if any, could you use to prove that the triangles are similar?

(a)



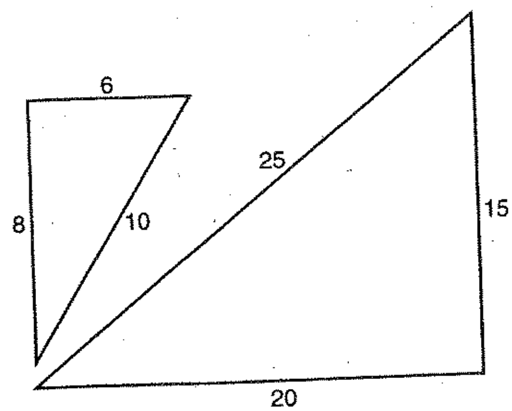
$\frac{3}{4} \stackrel{?}{=} \frac{12}{16}$

$3 \times 16 = 4 \times 12$

$48 = 48$

SAS Postulate

(b)



$\frac{6}{15} \stackrel{?}{=} \frac{10}{25}$

$6 \times 25 = 15 \times 10$

$150 = 150$

SSS Postulate

$\frac{6}{15} \stackrel{?}{=} \frac{8}{20}$

$6 \times 20 = 15 \times 8$

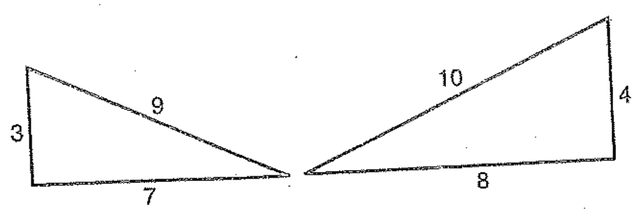
$120 = 120$

$\frac{8}{20} \stackrel{?}{=} \frac{10}{25}$

$8 \times 25 = 20 \times 10$

$200 = 200$

(c)



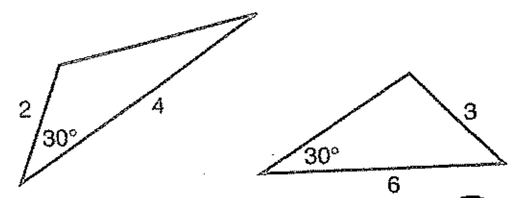
$\frac{3}{4} \stackrel{?}{=} \frac{9}{10}$

$3 \times 10 = 4 \times 9$

$30 \neq 36$

none

(d)



$\frac{2}{3} \stackrel{?}{=} \frac{4}{6}$

$2 \times 6 = 3 \times 4$

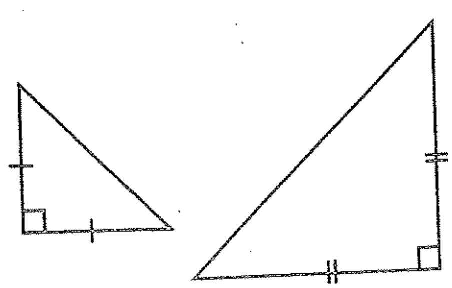
$12 = 12$, but the included $\angle$'s are not $\cong$

none

PRACTICE

Which postulate, if any, could you use to prove that the triangles are similar?

17.



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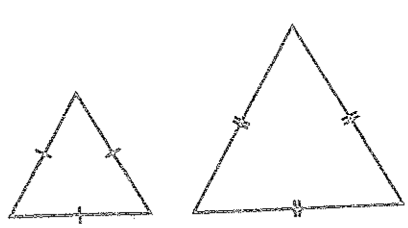

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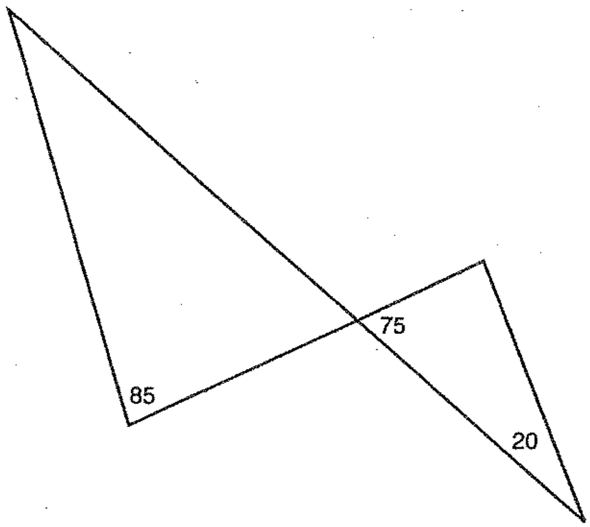

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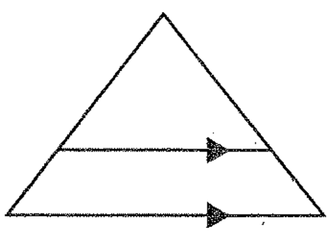

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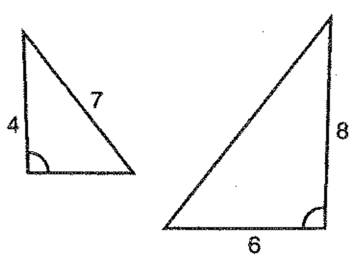

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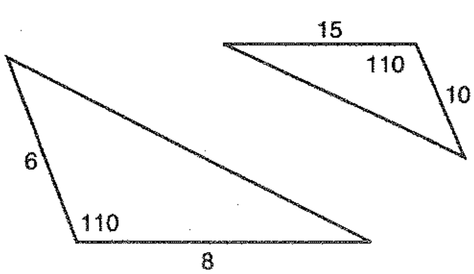

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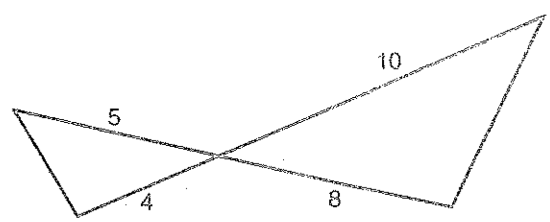

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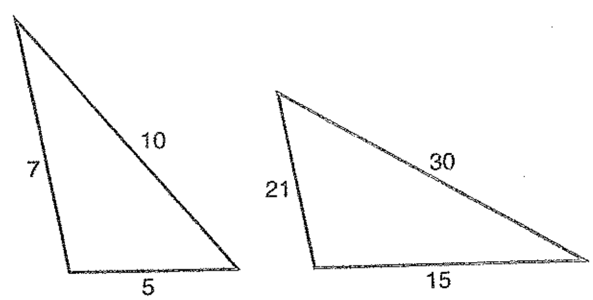

SOLVING FOR MISSING INFORMATION WITH SIMILAR TRIANGLES

25. Refer to Illustration 2, in which $\triangle ABC \cong \triangle DEF$.

a. Find $m(\overline{DE})$. **b.** Find $m(\angle E)$.

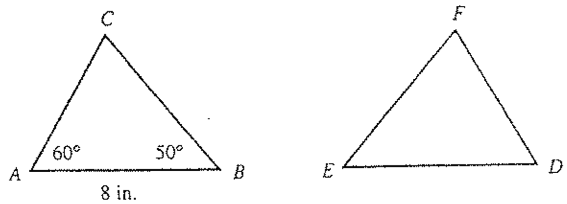

ILLUSTRATION 2

26. Refer to Illustration 3. Find x and y.

a. Find x. **b.** Find y.

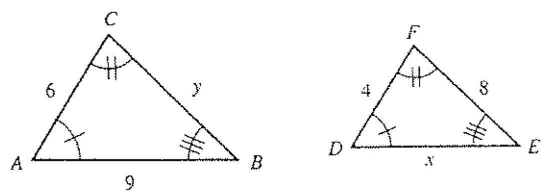

ILLUSTRATION 3

27. SHADOWS If a tree casts a 7-foot shadow at the same time as a man 6 feet tall casts a 2-foot shadow, how tall is the tree?

Answer Key

25. a. 8 in b. 50°

26. a. 6 b. 12

27. 21 ft.

13. yes
14. no
15. yes
16. yes
17. SAS
18. AA, SAS, or SSS
19. AA
20. AA
21. none
22. SAS
23. SAS
24. SSS