Key

POLYHEDRA WORKSHEET

Definitions:

Polyhedra (singular: polyhedron) : a closed surface made up of flat planar regions (called polygons) Polyhedra are solids.

Face: each flat planar region of a polyhedron is called a face

Edge: the place where two faces are joined is called an edge

Vertices: (singular: vertex) the place where three or more edges meet is called a vertex

Equilateral: the sides of the polygon are all congruent

Exploring the polyhedra:

1. Which of the shapes A – K meet the criteria in each part?
   a. The base is a equilateral pentagon G
   b. All the triangular faces are congruent A, C, H (G)(J)(M)(D)
   c. None of the triangular faces is equilateral G, I, J
   d. Two faces are parallel and congruent, but are not oriented the same way so the shape is not a prism J, K (H)
   e. All of the edges are congruent A, H, E (L), (M)
   f. No pair of edges are congruent all shapes have same congruent edges

2. The **prisms** are shapes B, C, E, and F. What common characteristics do they share? Write a definition for prism.

   2 congruent parallel faces (any shape) called <u>bases</u>. All other faces are parallelograms (including special parallelograms like rectangles). named by shape of base

3. The **pyramids** are shapes A, D, G, and I. What common characteristics do they share? Write a definition for pyramid.

   1 <u>base</u> (any shape), all other faces are triangles which meet at a vertex, called <u>apex</u>. named by shape of base

Definitions:
Prism: A convex polyhedron having two bases that are congruent and parallel (any shape polygon), and all the other faces (lateral faces) are parallelograms. Prisms are named by the shape of the base.

Pyramid: A convex polyhedron that has one face (any shape polygon) called a base and all other faces (lateral faces) are triangles. The vertex where the triangular faces meet is called an apex. Pyramids are named by the shape of the base.

Lateral edges: edges not along a base

Lateral faces: are faces that are not bases
*(Note: **all bases are faces**)*

4. Lateral edges and faces:
   a. For each prism and pyramid, tell how the number of lateral edges appear to be related to the edges on the base.

      # of lateral edges = # of base edges

   b. What shape are all the lateral faces on a prism? On a pyramid?

      prism: parallelograms & pyramid: triangles

   c. How many lateral edges and lateral faces does a 50-gonal pyramid have? How many lateral faces? 50 lateral edges
      50 lateral faces

Right prism: A prism where the lateral edges make right angles with (or are perpendicular to) the base edges and all lateral faces are rectangles (it is called an **oblique prism** otherwise).

Regular pyramid: A pyramid that has a regular polygonal region as a base, with all other faces congruent isosceles triangles and the apex is centered over the base (it is called an **oblique pyramid** otherwise).

5. Identifying
   a. Which, if any, of the shapes are right prisms?

      C, E, F

   b. Which, if any, of the shapes are oblique prisms?

      B

   c. Which, if any, of the shapes are regular pyramids?

      A, G

   d. Are all the lateral edges of a regular pyramid the same length?

      yes

6. Using the name of the base and the vocabulary of right, regular, and oblique, name the shapes:

A: regular triangular pyramid (regular tetrahedron)

B: oblique parallelogramal prism (parallopiped)

C: right triangularprism

D: oblique rectangular pyramid

E: right square prism (cube)

F: right rectangular prism

G: regular pentagonal pyramid

I: oblique square pyramid

(J: antiprism) (K: prismoid)

RELATED BONUS QUESTIONS:

7. What geometric name best applies to each of these?
   a. A shoebox right rectangular prism
   b. an unsharpened pencil with flat sides (and no eraser) right hexagonal prism
   c. an unused eraser (not on a pencil) oblique rectangular prism

8. Who am I? exercises are common n elementary curriculum. Ideally clues are revealed one at a time, each clue is discussed, until the answer is obtained. Here is an example.

I am a polyhedron.
I have 7 faces.
Six of my lateral edges are equal in length.
One of my faces is a hexagon.
Who am I?

regular hexagonal pyramid

DEVELOPING EULER'S FORMULA

Developing Eulers formula:

a. Find all the pyramids you have in shapes A – K. Count the number of faces (F), the number of vertices (V), and the number of edges (E) for each one. Organize the data and look for patterns or relationships.

	A	D	I	G
F	4	5	5	6
V	4	5	5	6
E	6	8	8	10

b. Using the patterns you noted above, find the number of F, V and E for a *hexagonal* pyramid, a *100-gonal* pyramid, an *n-gonal* pyramid and a *(x+y)-gonal* pyramid. Do the patterns and relationships hold for these pyramids also?

$$\left.\begin{aligned} F &= n+1 \\ V &= n+1 \\ E &= 2n \end{aligned}\right\} n\text{-gonal}$$

c. Find all the prisms in your shapes A – K. Count the number of faces (F), vertices (V) and edges (E) for each one. Organize the data and look for patterns or relationships.

	B	C	E	F
F	6	5	6	6
V	8	6	8	8
E	12	9	12	12

d. Find the number of F, V and E for a hexagonal prism, a 100-gonal prism, a n-gonal prism and a (x+y)-gonal prism. Do the patterns and relationships hold for these prisms also?

$$\left.\begin{aligned} F &= n+2 \\ V &= 2n \\ E &= 3n \end{aligned}\right\} n\text{-gonal}$$

For pyramids: You should have noted the following-

- The number of faces equals the number of vertices
- The number of faces and vertices is always one more than the number of sides on the base
- The number of edges is equal to 2 times the number of sides on the base

For prisms: You should have noted the following-

- The number of faces is equal to the number of sides on the base plus 2
- The number of vertices is equal to double the number of sides on the base
- The number of edges is equal to 3 times the number of sides on the base

One relationship you may have overlooked *relates all three quantities, F, V, and E* and works for both the pyramids and prisms. If you did not see such a relationship, look for one now. This relationship is called ***Eulers formula***.

$$F + V - 2 = E$$

e. Test the relationship you found in part f with shapes H, J, and K and any other polyhedra you may have.

This works for <u>any</u> polyhedron

RELATED BONUS QUESTIONS

1. Suppose a polyhedron has 10 vertices and 15 edges. How many faces does it have?

$$F + 10 - 2 = 15$$
$$F = 7$$

2. Suppose another polyhedron has 20 faces and 30 edges. How many vertices does it have?

$$20 + V - 2 = 30$$
$$V = 12$$

3. Can the number of vertices, the number of faces, and the number of edges of a polyhedron all be odd numbers? Why or why not?

no, all cannot be odd, if F is odd and V is odd, then F + V − 2 is even.

<u>etc</u>

4. React to the following scenarios:

a. A student says, "I was thinking. A square pyramid has 4 triangular faces and a square base. Each triangular face has 3 sides, which are edges of the pyramid. So there should be 4 times 3, or 12, lateral edges plus the 4 edges on the square base. That is 16 edges in all. But I only counted 8 edges." What is the matter with the students reasoning?

edges are shared by two faces, so double counting.

etc.

b. A student says, "I know that a cube has 6 square faces, and a square has 4 vertices. So it seems that there should be 6 times 4, or 24 vertices." What is the matter with the students thinking?

verticies are shared by three faces, so triple counting.

etc.