4.1.1 Translating and Solving Equations

Example 1 Translate and Solve

a. Translate “two times the sum of a number and eight equals the sum of four times the number and six” into an equation and solve.

b. Translate “nine less than twice a number is five times the sum of the number and twelve” into an equation and solve.

Defining Variables, Writing and Solving Equations

Example 2 Defining Variables, then Writing and Solving Equations

The temperature of the sun on the Kelvin scale is 6500 degrees Kelvin. This is 4740 less than the temperature on the Fahrenheit scale. Find the Fahrenheit temperature.

a. Define the variable (the unknown).

b. Write an equation.

c. Solve the equation.
Example 3  Defining Variables, then Writing and Solving Equations
A board 10 ft long is cut into two pieces. Three times the length of the shorter piece is twice the length of the longer piece. Find the length of each piece.

a. Define the variable and express each unknown in terms of the variable.

b. Write an equation.

c. Solve the equation.
   Answer the question.

Example 4  Defining Variables, then Writing and Solving Equations
A company manufactures 160 bicycles per day. Four times the number of 3-speed bicycles made equals 30 less than the number of 10-speed bicycles made. Find the number of 10-speed bicycles manufactured each day.

a. Define the variable and express each unknown in terms of the variable.

b. Write an equation.

c. Solve the equation.

d. Answer the question.
Example 5  Defining Variables, then Writing and Solving Equations
A union charges monthly dues of $4.00 plus $0.15 for each hour worked during the month. A union member’s dues for March were $29.20. How many hours did the union member work during the month of March?

a. What is the unknown?
   Define the variable.

b. Write an equation.

c. Solve the equation.

d. Answer the question
   (include units).

Example 5  Defining Variables, then Writing and Solving Equations
Each month union members pay $4 plus 1.5% of their salary. A union member’s dues for March were $49. What was the union worker’s salary in March?
4.2.2 Consecutive Integer Problems

Consecutive Integers
a. Write 3 consecutive positive integers.  
   \[7, 8, 9\]

b. Write 3 consecutive negative integers.

c. Write a variable representation for 3 consecutive integers.

Consecutive Odd Integers
a. Write 3 consecutive odd positive integers.

b. Write 3 consecutive odd negative integers.

c. Write a variable representation for 3 consecutive odd integers.

Consecutive Even Integers
a. Write 3 consecutive even positive integers.

b. Write 3 consecutive even negative integers.

c. Write a variable representation for 3 consecutive even integers.
Example 1
The sum of three consecutive odd integers is 51. Find the three integers.

a. Define a variable and express each integer in terms of that variable.

b. Write and equation and solve. Answer the question.

Example 2
Find three consecutive even integers such that three times the second is six more than the sum of the first and third.

b. Define a variable and express each integer in terms of that variable.

b. Write and equation and solve. Answer the question.

Example 3
Find three consecutive integers whose sum is 12.
4.2.2 Coin and Stamp Problems

Write an expression and find the value (in cents) of each of the following:

a. three quarters
b. seven dimes
c. \(n\) nickels
d. four thirty-three cents stamps
e. \(x\) eight cent stamps

**Coin and Stamp Equation**

\[
\begin{pmatrix}
\text{Number of Stamps} \\
\text{Unit Value of each Stamp}
\end{pmatrix}
\cdot
\begin{pmatrix}
\text{Unit Value of each Stamp}
\end{pmatrix}
= \begin{pmatrix}
\text{Total Value of Stamps}
\end{pmatrix}
\]

\[N \cdot U = V\]
Example 1  Coin Problem
A coin bank contains $1.20 in dimes and quarters. In all, there are nine coins in the bank. Find the number of dimes and quarters in the bank.

a. What are the unknowns?
Define one of the unknowns to be the variable and express the other unknown in terms of that variable.

b. Organize the data in a table.

<table>
<thead>
<tr>
<th>Stamp or Coin</th>
<th>Number $N$</th>
<th>Unit Value $U$</th>
<th>Total Value $V$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$=$</td>
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<td>$=$</td>
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<tr>
<td>Total</td>
<td></td>
<td>$=$</td>
<td>$=$</td>
</tr>
</tbody>
</table>

c. Write and solve an equation that expresses the relationship of the values of the coins. Answer the question.
Example 2   Stamp Problem
A collection of stamps consists of 3-cent stamps and 8-cent stamps. The number of 8-cent stamps is five more than three times the number of 3-cent stamps. The total value of the stamps is $1.75. Find the number of each type of stamp.

a. What are the unknowns?

b. Organize the data in a table. Write and solve an equation. Answer the question.

<table>
<thead>
<tr>
<th>Stamp or Coin</th>
<th>Number ( N )</th>
<th>Unit Value ( U )</th>
<th>Total Value ( V )</th>
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<td>Total</td>
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</table>

Example 3   Coin Problem
A coin bank contains $6.30 in nickels, dimes and quarters. There are five times as many nickels as dimes and six more quarters than dimes. Find the number of each kind of coin in the bank.
4.3.1 Perimeter Problems

The perimeter, denoted $P$, of a closed geometric plane figure is the measure of the distance around the figure.

**Triangles**

Triangles with side lengths $a$, $b$, and $c$ has perimeter $P = a + b + c$.

An **isosceles** triangle with two sides of equal length $a$ has perimeter $P = 2a + b$.

An **equilateral triangle** with all three sides of equal length $a$ has perimeter $P = 3a$.

**Rectangles**

A rectangle with length $L$ and width $W$ has perimeter $P = 2L + 2W$.

A **square** with side length $x$ has perimeter $P = 4x$. 
Example 1
The perimeter of a rectangle is 26 feet. The length of the rectangle is 1 foot more than twice the width. Find the width and length of the rectangle.
a. Draw a picture.

b. Write the perimeter formula and substitute in all known values. Then solve for the remaining variable. Answer the question.

Example 2
The perimeter of an isosceles triangle is 25 feet. The length of the third side is 2 feet less than the length of one of the equal sides. Find the length of each side of the triangle.
a. Draw a picture and label all sides.

b. Write the perimeter formula and substitute all known values. Then solve for the remaining variable. Answer the question.
Example 3
A carpenter is designing a square patio with a perimeter of 52 feet. What is the length of each side?

a. Draw a picture and label all sides.

b. Write the perimeter formula and substitute all known values. Then solve for the remaining variable.
4.3.2 Angles of Intersecting Lines

Terminology of Angles
1. The units for measuring angles are **degrees**, where one complete revolution is 360°.

2. The symbol for an angle is \( \angle \). That is, \( \angle a \) is read “angle a.”

3. A 90° degree angle is called a **right angle**.

4. A 180° degree angle is called a **straight angle**.

5. An **acute angle** is an angle whose measure is between 0° and 90°.

6. An **obtuse angle** is an angle whose measure is between 90° and 180°.

7. **Complementary angles** are two angles whose sum is 90°.

8. **Supplementary angles** are two angles whose sum is 180°.
Example 1
a. Find the complement of a $30^\circ$ angle.

b. Find the supplement of a $110^\circ$ angle.

**Parallel and Perpendicular Lines**
**Parallel lines** never intersect.

**Perpendicular lines** intersect at right angles.
General Intersecting Lines
When two non-perpendicular lines intersect four angles are formed - two are acute and two are obtuse.

Vertical angles are angles that lie on opposite sides of two intersecting lines. Vertical angles have equal measures.

Adjacent angles have a common side. Adjacent angles of intersecting lines are supplementary angles.

Example 1
Refer to the figure above.

a. List two acute vertical angles. What can be said about the measure of the two angles?

b. List two obtuse vertical angles. What can be said about the measure of the two angles?

c. List four pair of adjacent angles. What is the sum of the measure of each pair of adjacent angles.
Definitions and Questions for Understanding

1. A line that intersects two parallel lines is called a transversal line.

2. Alternate interior angles are two angles that are on opposite sides of the transversal line, inside the parallel lines, and are both acute or both obtuse. **Alternate interior angles have the same measure.** List all pairs of equal alternate interior angles.

3. Alternate exterior angles are two angles that are on opposite sides of the transversal line, outside the parallel lines, and are both acute or both obtuse. **Alternate exterior angles have the same measure.** List all pairs of equal alternate exterior angles.

4. Corresponding angles are two angles that are on the same side of the transversal line and are both acute or both obtuse. **Corresponding angles have the same measure.** List all pairs of equal corresponding angles.

Angles in Triangles

The sum of the measures of the interior angles of a triangle is 180°.

\[ \angle a + \angle b + \angle c = 180^\circ \]
4.4 Markup and Discount

4.4.1 Markup Problems

Markup Equation Variables
1. The cost, $C$, is the price a business pays for a product.
2. The selling price, $S$, is the price a business sells a product for.
3. The markup, $M$, is the difference between the selling price and the cost ($M = S - C$). Often the markup is a percent of the cost. That is, $M = rC$, where $r$ is the markup rate in decimal form.

Selling Price = Cost + Markup: \[ S = C + M \]
Markup = (Markup rate) · (Cost) \[ M = r \cdot C \]

**Markup Equation** \[ S = C + r \cdot C \]

Example 1
The manager of a clothing store buys a suit for $90 and sells it for $126.
a. Find the markup rate.  
b. Find the markup.

Example 2
The cost to a sporting goods store for a tennis racket is $60. The selling price of the racket is $90.
a. Find the markup rate.  
b. Find the markup.
Example 3
The manager of a furniture store uses a markup rate of 45% on all items. The selling price of a chair is $232. Find the cost of the chair.

Example 4
A hardware store uses a markup rate of 40% on all items. The selling price of a lawnmower is $133. Find the cost.
4.4.2 Discount Problems

Sale Price = Regular Price - Discount \[ S = R - D \]
Discount = (Discount rate) \times (Regular price) \[ D = R \cdot r \]

Discount Equation
\[
S = R - R \cdot r \\
S = R(1 - r)
\]

Example 1
The regular price for a 100 ft garden hose is $48. During a sale the hose is being sold for $36.

a. Find the discount rate. 

b. Find the discount.

Example 2
A case of motor oil that regularly sells for $29.80 is on sale for $22.35.

a. Find the discount rate? 

b. Find the discount.
Example 3
The sale price for a chemical sprayer is $27.30. This is 35% off the regular price. Find the regular price.

Example 4
The sale price for a telephone is $43.50. This is 25% off the regular price. Find the regular price.
4.5 Investment Problems

**Simple Interest Formula:** \[ I = Principa \cdot rate \]
\[ I = P \cdot r \]

where \( I \) = annual simple interest earned,
\( P \) = amount of principal invested, and
\( r \) = annual simple interest rate in decimal number form

**Example 1**
Find the annual simple interest earned on a $4500 investment at an 8% annual simple interest rate.

**Example 2**
An investor has a total of $10,000 to deposit into two simple interest accounts. One account earns an annual simple interest rate of 7%, and the other earns 8%. How much should be invested in each account so that the total annual interest earned is $785?

<table>
<thead>
<tr>
<th>Investment</th>
<th>Principal ( P )</th>
<th>Rate ( r )</th>
<th>Interest Earned ( I )</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Total</td>
<td>( \cdot )</td>
<td>=</td>
<td></td>
</tr>
</tbody>
</table>
Example 3
A dentist invested a portion of $15,000 in a 7% simple interest account and the remainder in a 6.5% simple interest government bond. The two investments earn $1020 in interest annually. How much was invested in each account?

Example 4
An investment counselor invested 75% of a client’s money into a 9% annual simple interest rate money market fund. The remainder was invested in 6% annual simple interest government securities. Find the amount invested in each if the total interest earned is $3300.
Example 5
An investment banker invested 55% of the bank’s available cash in an account that earns 8.25% annual simple interest. The remainder of the cash was placed in an account that earns 10% annual simple interest. The interest earned in one year was $58,743.75. Find the total amount invested.

Example 6
An investment of $2500 is made at an annual simple interest rate of 7%. How much additional money must be invested at 10% so that the total interest earned will be 9% of the total investment?
Example 7
A charity deposited a total of $54,000 into two simple interest accounts. The annual simple interest rate on one account is 8%. The annual simple interest rate on the second account is 12%. How much was invested in each account if the total annual interest earned is 9% the total investment?
4.6.1 Value Mixture Problems

What is the value of 5 oz of a gold alloy that cost $185 per ounce?

**Value Mixture Equation**

\[ \text{Amount} \cdot \text{Unit Cost} = \text{Value} \]

\[ A \cdot C = V \]

**Example 1**

A coffee merchant wants to make 9 lb of a blend of coffee costing $6 per pound. The blend is made using a $7 grade and a $4 grade of coffee. How much of each grade should be used?

<table>
<thead>
<tr>
<th>Mixture</th>
<th>Amount</th>
<th>Unit Cost</th>
<th>Value</th>
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</thead>
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<td>Total</td>
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Example 2
How many ounces of a silver alloy that costs $6 an ounce must be mixed with 10 oz of a silver alloy that costs $8 an ounce to make a mixture that cost $6.50 an ounce?

Example 3
A gardener has 20 pounds of lawn fertilizer that cost $0.90 per pound. How many pounds of a fertilizer that cost $0.75 per pound should be mixed with this to produce a mixture that cost $0.85 per pound?
4.6.2 Percent Mixture Problems

Take Note
a. A 5% saltwater solution means that 5% of the total solution is pure salt and the remaining 95% of the solution is water and other fluids.

b. A 27% silver alloy means that 27% of the metal is pure silver and the remaining 73% of the metal is other stuff.

Percent Mixture Equation

\[ Amount \cdot Concentration = Quantity \]

\[ A \cdot C = Q \]

Example 1
A 125 ml bottle contains a 3% hydrogen peroxide solution. Find the amount of pure hydrogen peroxide in the solution.
**Example 2**
How many gallons of a 15% salt solution must be mixed with 4 gal of a 20% salt solution to make a 17% salt solution?

a. The substance pure __________ is being isolated.

b. Organize the data and solve the problem.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Amount $A$</th>
<th>Concentration $C$</th>
<th>=</th>
<th>Quantity $Q$</th>
</tr>
</thead>
<tbody>
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<tr>
<td>Total</td>
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<td></td>
</tr>
</tbody>
</table>
Example 3
How many quarts of pure orange juice must be added to 5 quarts of a fruit drink that is 10% orange juice to make an orange drink mix that is 25% orange juice?

a. The substance pure __________ is being isolated.

b. Organize the data and solve the problem.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Amount $A$</th>
<th>Concentration $C$</th>
<th>=</th>
<th>Quantity $Q$</th>
</tr>
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Example 4
A recipe for a rice dish calls for 12 oz of a rice mixture that is 20% wild rice and 8 oz of pure wild rice. What is the percent concentration of wild rice in the mixture?

<table>
<thead>
<tr>
<th>Solution</th>
<th>Amount (A)</th>
<th>Concentration (C)</th>
<th>=</th>
<th>Quantity (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>\cdot</td>
<td>\cdot</td>
<td>=</td>
<td>\cdot</td>
</tr>
<tr>
<td>B</td>
<td>\cdot</td>
<td>\cdot</td>
<td>=</td>
<td>\cdot</td>
</tr>
<tr>
<td>Total</td>
<td>\cdot</td>
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</tr>
</tbody>
</table>
4.7 Uniform Motion Problems

Uniform motion is motion at a constant speed in a constant direction.

**Uniform Motion Equation**

\[ Rate \cdot Time = Distance \]
\[ R \cdot T = D \]

**Example 1**

A car leaves a town traveling 35 mph. Two hours later, a second car leaves the same town, on the same road, traveling in the same direction at 55 mph. How many hours after the second car leaves will the second car pass the first car?

a. Do the two cars start at the same place? Does the experiment end when the two cars end at the same place?

b. Organize the data in the table.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Rate</th>
<th>Time</th>
<th>=</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>=</td>
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<td>Total</td>
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</table>

c. Write an equation that describes the relationship between the distances of the two cars. Solve the equation and answer the question.
Example 2
Two cars, one traveling 10 mph faster than a second car, start at the same time from the same point and travel in opposite directions. In 3 hours, they are 288 miles apart. Find the rate of the second car.

a. Organize the data in the table.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Total</td>
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</tbody>
</table>

b. Write an equation that describes the relationship between the distances of the two cars. Solve the equation and answer the question.

Example 3
Two trains, one traveling at twice the speed of the other, start at the same time from stations that are 306 miles apart and travel toward each other. In 3 hours, the trains pass each other. Find the rate of each train.
Example 4
On a survey mission a pilot flew to a parcel of land and back in 7 hours. The rate out was 120 mph. The rate back was 90 mph. How far away was the parcel of land?

Example 5
A bicycling club rides out into the country at a speed of 16 mph and returns over the same road at 12 mph. How far does the club ride into the country if it travels a total of 7 hours?
4.8 Applications of Inequalities

Example 1
a. What is the minimum integer that satisfies the inequality
   \[ x > -14. \]

b. What is the maximum integer that satisfies the inequality
   \[ x < 8. \]

Example 2
If a rental car cost $15 per day and 20 cents per mile.

a. What is the unknown quantity in this problem. Define the variable.

b. Write an expression for the total cost of the car for one day.

c. Write an expression for the total cost of the car for one week.
Example 3
A student must have at least 450 points out of 500 points on five tests to receive an A in a course. One student’s results on the first four tests were 93, 79, 87, and 94. What scores on the last test will enable this student to receive an A in the course?

a. Let \( x = \) _______________________

b. Write and solve an inequality. Answer the question.

Example 4
An appliance dealer will make a profit on the sale of a television set if the cost of the new set is less than 70% of the selling price. What minimum selling price will enable the dealer to make a profit on a television set that costs the dealer $340?
Example 5
The base of a triangle is 8 inches and the height is \((3x + 5)\) inches. Express as an integer the maximum height of the triangle when the area is less than 112 inches.

Example 6
Company A rents cars for $9 and 10 cents per mile driven. Company B rents cars for $12 per day and 8 cents per mile driven. If your rent a car for one week what is the maximum number of miles you can drive a Company A car if it is to cost you less than Company B car?